Always assume that we are not trying to divide by a zero probability:

1. Prove or disprove:

If \( P[A|\bar{B}] = P[A|B] \), then \( A \) and \( B \) are independent.

2) Prove that \( \text{cov}( \sum_i a_i X_i, \sum_j b_j Y_j ) = \sum_i \sum_j a_i b_j \text{cov}(X_i, Y_j) \)
3) Assume there are 5 stocks, each of which sells for $100 per share and has the same expected annual return per share, $\mu$, and the same variance of return, $\sigma^2$. Assume the returns on the five stocks are pairwise independent.

(a) if you buy ten shares of one stock, what will be the mean and variance of the annual return on your portfolio?

(b) What if you buy two shares of each stock?

(c) Now assume that 16 of the covariances are -1 and the rest are 1. Recompute your answers for (a) and (b).
4) Assume the linear model where \( x \) is a \((1 \times t)\) vector and \( u \) is a scalar. Use LIE to prove that \( E(u|x) = 0 \) implies

(a) \( E(u) = 0 \)

(b) \( \text{cov}(x, u) = 0. \)

(c) Which of these assumptions is true for the following diagram?
5. Let $X, Y,$ and $Z$ be random variables, each of which takes only two values: 0 and 1.

Given $P(X = 1) = .5, P(Y = 1|X = 1) = .6, P(Y = 1|X = 0) = .4, P(Z = 1|Y = 1) = .7, P(Z = 1|Y = 0) = .3,$ find $E Z$.

6) Compute $\text{cov}(3X + 4Y - Z, X - 2Z)$ assuming that $\sigma_X^2 = \sigma_Z^2 = 2, \sigma_{X,Z} = 3,$ and $\sigma_{Y,X} = \sigma_{Y,Z} = 4$. 
7) Assume the following discrete distribution for $X$ and $Y$:

a. Now fill in the tables for $f(y|X = 1)$ and $f(y|X = 2)$

b. Compute $f_Y(y)$ and $f_X(x)$

c. Compute $E[E(Y|X)]$

d. Compute $E[f(1|X)]$