Homework 3.

1. Suppose there exists a single seller of health insurance and a population of 100 potential buyers. One consumer has an expected claim of $100, one has an expected claim of $200, one with $300, one with $400, etc., and one has an expected claim of $10,000, so that they are evenly distributed. The seller does not know what an individual buyer’s claim is and therefore cannot price discriminate. By buying insurance, each buyer receives a risk-reduction benefit of $500.

   a. What is the most that a consumer with an expected claim of $7,000 will be willing to pay for insurance?

   b. Let $x$ be a consumer’s expected claim. Derive a formula that gives the maximum price that consumer will be willing to pay as a function of $x$.

   c. If the insurer sets a price equal to the maximum price that the consumer with an expected claim of $7,000 is willing to pay, which consumers will buy insurance? What will the average claim be among these consumers?

   d. Given that all consumers with expected claims bigger than or equal to $x$ buy insurance, what is the minimum price that the insurance company must charge these customers (assume that there is a consumer with claim value $x$, that is, $x$ is an integer multiple of 100 and is no larger than 10,000) in order to break even.

   e. If the price for insurance is set as in part c, will the insurance company lose money, break even or make a profit? What is its (loss/profit)?

   f. Derive the price for insurance at which some consumers are willing to purchase insurance and the insurance company exactly breaks even.

   g. Now suppose that the insurance company also faces an administrative cost, given by the variable $c$, that it must pay for each dollar in claims that it pays. For which values of $c$ is it impossible to find a price, as in part f, such that the insurance company breaks even and some consumers are willing to buy insurance.
2. Karen Milgrom is the CEO and sole owner of MightySoft, which she founded. Although she is quite adept as a manager, she nonetheless spends some company money on perquisites for herself, including a nicer-than-necessary company apartment in Manhattan (The total amount of perquisites is $5 million). Suppose that, if Karen sold all of her interest in Mightysoft to rival Bill Gates and Gates took over as manager, Mightysoft would sell for $100 million (Gates’ preferences are identical to Karen’s and each knows what the other’s preferences are). This is represented in the graph below.

Suppose Karen sells Gates 50% of the company but stays on as CEO. What would happen to the value of the firm? Demonstrate this in a diagram.

3. Suppose Professor Turner has the following utility function

\[ U(x) = 1 - e^{-0.00002x}, \]

where \( x \) gives his total assets. He is offered the following three gambles:

- Gamble A pays $50,000 with certainty
- Gamble B pays $100,000 with probability .8 and $0 with probability .2
- Gamble C pays $200,000 with probability .7 and $0 with probability .3

These winnings are added to Professor Turner's assets, which at the moment are $500,000. So, for instance, if Professor Turner chooses Gamble A, his (expected) utility is \( 1 - e^{-0.00002(550,000)} \).

a. Compute the expected utility of each gamble and identify the risk premia and certainty equivalents. Which does Professor Turner prefer?

b. Suppose an additional "gamble" Z exists that pays $Z for certain. How large does Z need to be in order for Professor Turner to prefer Gamble Z over Gamble B?