Homework 4 Answers.

This problem addresses some, but not all, of the issues covering incentive contracting. Notably, it does not address the risk/incentive tradeoff. In the interest of helping you understand this, I address what would happen if the agent were risk averse in this answer key.

a. The manager's expected benefit is $200T\beta$, while his cost is $T^2$. Equating marginal benefit to marginal cost, we find:

$$T = 100\beta$$

gives the optimal number of transactions.

b. This question assumes that the principal's (Enran) expected payoff is the company profit, $200T$, minus the wage paid to the division manager, $W_0 + \beta 200T$. By substituting $T = 100\beta$ in for $T$, we get:

$$\text{Expected Payoff} = 20,000(\beta - \beta^2) - W_0$$

c. It is easy to show that $\beta=0.5$ maximizes the above expression. It does not maximize total value, which is:

$$200T - T^2,$$

or, substituting in $\beta$,

$$20,000\beta - 10,000\beta^2.$$

This is maximized by setting $\beta=1$. The intuition for why $\beta=0.5$ is inefficient is that the agent is paying the full cost of effort, but is only receiving a $\beta$ share of the profit. In order to induce the efficient effort $\beta=1$ is optimal.

In this problem, when the efficient level of incentives are provided, the principal's payoff is $-W_0$, so that in essence the agent is paying the principal for the right to be a division manager. This is a quirk of the problem that stems from the assumption of risk neutrality. So consider the case where the agent is risk averse, with risk tolerance $r$. Then the risk premium,

$$\frac{1}{2} r\beta^2 \text{Var}(\psi) = \frac{1}{2} r [C'(T)]^2 \text{Var}(\psi)$$
enters the total certain equivalent. To find the optimal $T$ and $\beta$, we must write the risk premium with the substitution $200\beta = C'(T) = 2T$. Then the total certain equivalent becomes:

$$200T - T^2 - \frac{1}{2} r\left(\frac{T}{100}\right)^2 \text{Var}(\psi).$$

Maximizing this with respect to $T$, we have

$$200 - 2T - \frac{rTVar(\psi)}{10,000} = 0$$

Then, substituting $T = 100\beta$ back into the expression, we have

$$200 - 200\beta - \frac{rVar(\psi)}{100} \beta = 0.$$  
This yields

$$\beta = \frac{200}{200 + \frac{rVar(\psi)}{100}}.$$  
The right-hand side of this is clearly smaller than 1 whenever $rVar(\psi)$ is positive, so if the agent is risk averse and there is some risk to being a division manager, then it is optimal to lower the incentives accordingly.

Compare this solution for the optimal $\beta$ to the formula for the Incentive Intensity Principal from the book and the last day of class:

$$\beta = \frac{P'(e)}{1 + rC''(e)\text{Var}(x + \gamma)}.$$  
If we think of effort as being represented by transactions, then we can relate these two expressions with a bit of work. It is clear that the analog of $P'(e)$ here is 200, as the company's profit increases by 200 with each transaction. In the problem in the book, the agent was only rewarded $\beta$ for each unit of effort. Here, by contrast, she gets $200\beta$. So the "1" in the denominator of the above expression is therefore replaced by a "200."

The final piece is the $C''(e)$ term, and the $200\beta$ compensation comes up again. Since, in equating marginal benefits to marginal cost, the agent sets $C'(T) = 200\beta$, we have that
\[ \beta = \frac{C'(T)}{200} \]

The derivative of this term,

\[ \frac{C''(T)}{200} = \frac{1}{100}, \]

is what shows up in the expression

\[ \beta = \frac{200}{200 + \frac{rVar(\psi)}{100}}. \]

\[ d. \] Under risk neutrality, this is very easy. With \( \beta = .5 \), the agent optimally executes 50 transactions. She gets a fixed component of 1,000, a "transaction" component of \(.5(10,000) = 5,000\), and suffers an effort cost of \(50(50) = 2,500\). So her expected payoff is 3,500, clearly better than the 2,500 outside option.

If she were risk averse, then her choice would factor in the risk premium,

\[ \frac{1}{2} r\beta^2 Var(\psi) = \frac{1}{2} r(.25)(5,500^2) = 3,781,250r. \]

If this is smaller than 1,000, then the agent will take the job. This occurs whenever

\[ r < .000264. \]