Appendix to "Trends in Private Patent Costs and Rents for Publicly-Traded United States Firms"

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This appendix includes two sections. First, we offer a detailed mathematical exposition of the verbal model from section 3 of the paper. This exposition includes the technical foundation for our claim that the event study understates the total costs of patent litigation. Second, we report additional estimates of cumulative abnormal returns (CARs) and aggregate litigation costs. These results serve as additional robustness checks on our estimates of litigation costs.

1. A Theoretical Model of Patent Litigation

Consider a patentee firm that owns a single patent and a single rival who may have infringed the patent. Let the likelihood the patentee would win an infringement suit be π. We will interpret this as an index of the patent's overall strength. Let any per-unit royalty paid by the alleged infringer to the patentee be r, and let any lump sum payment be T. Let all payoffs that do not depend on the patent be P_0 for the patentee and R_0 for the rival, and for simplicity normalize these payoffs to zero.

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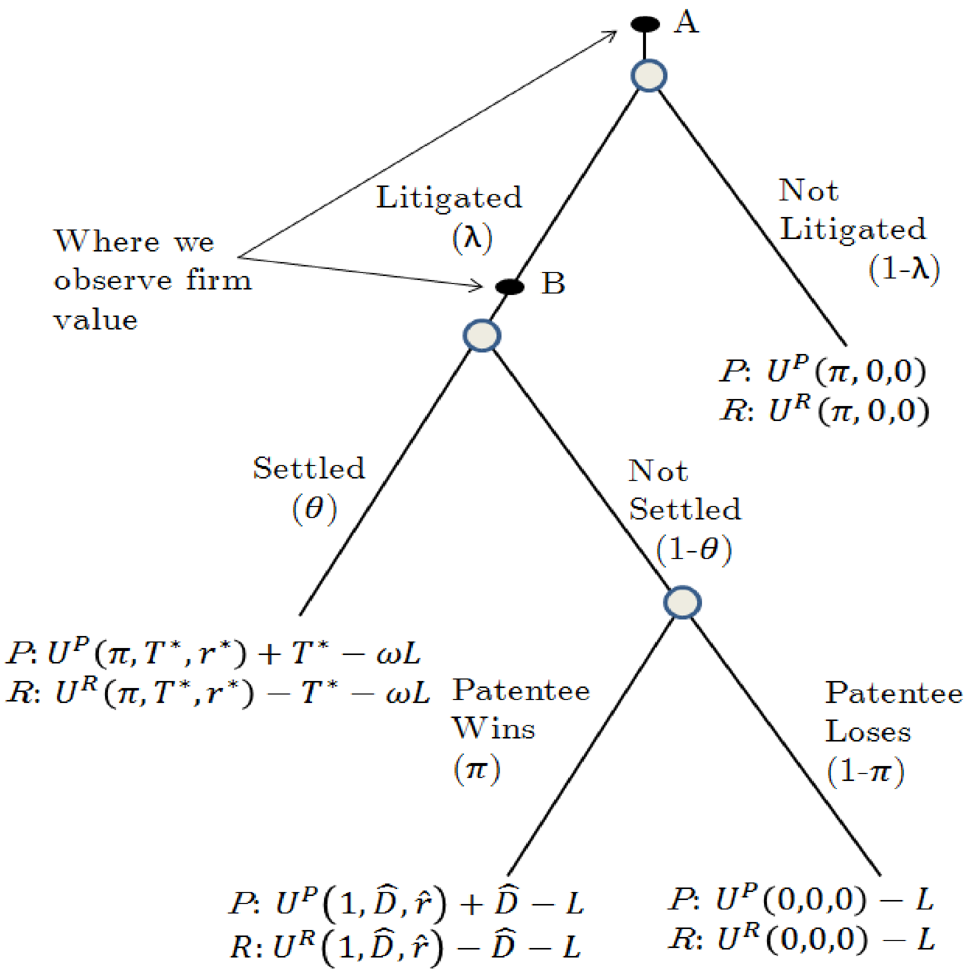

Figure 1: *Litigation Game Tree*

By virtue of having the patent, the patentee earns profits that are no lower than it otherwise would. Call these extra profits *rents* and denote them as $U^P(\pi, T, r)$. Intuitively, we will think of rents as depending upon the overall strength of the patent π, and on royalties T and r that obtain either due to settlement or due to a court decision. Note also that we define utility net of any transfer payments from the defendant, so it is best to think of them as all rents other than pure transfers from the defendant.

For the defendant, denote its rents as $U^R(\pi, T, r)$. The second and third arguments reflect the fact that other rivals may respond to the levels of T and r. Note that r captures all aspects of any bargain other than T, including whether there is a settlement.

Consider the game tree in Figure 1. At point A, litigation has not yet been filed. The probability that litigation occurs is λ. If litigation does not occur, then the payoffs are:

$$P : U^P(\pi, 0, 0)$$
$$R : U^R(\pi, 0, 0)$$

We denote these rents as *baseline*.

If litigation is filed, then the patentee and the alleged infringer bargain over a settlement in the shadow of possible litigation. For simplicity we restrict attention to one round of 50/50 Nash bargaining followed (possibly) by one simple litigation subgame. If the firms do not settle, then each firm pays cost L for full-blown litigation that has two probabilistic outcomes. With probability π the patentee wins damages, where the court assigns payment $\widehat{D}$ and ongoing royalty rate $\widehat{r}$. The patentee owns an ironclad patent and earns maximal private rents. The payoffs are:

$$P : U^P(1, \widehat{D}, \widehat{r}) + \widehat{D} - L$$
$$R : U^R(1, \widehat{D}, \widehat{r}) - \widehat{D} - L$$

With probability $1 - \pi$ the patentee does not win damages, owns a worthless patent and receives no rents. The payoffs are:

$$P : U^P(0, 0, 0) - L$$
$$R : U^R(0, 0, 0) - L$$

Prior to litigation, the firms bargain over terms of a settlement. With probability θ, litigation costs under settlement equal ωL, with $\omega < 1$, and settlement is efficient. With probability $1 - \theta$, litigation costs under settlement are so large that settlement is inefficient and litigation ensues.

If the terms of the settlement are such that the alleged infringer agrees to pay royalties according to r and fixed payment T, then the payoffs are

$$P : U(\pi, T, r) + T - \omega L$$
$$R : U(\pi, T, r) - T - \omega L$$

In an equilibrium Nash bargain, the firms choose r and T to maximize their *joint* rents. Moreover, each firm earns its expected payoff under litigation (including expected damages) plus one half of the extra surplus earned from settlement versus litigation.

Litigation costs include direct costs L_D and indirect costs L_I:

$$L = L_D + L_I$$

In pursuing litigation, both the patentee and alleged infringer must commit direct costs to preparing for settlement negotiation and possible courtroom activity. These resources include routine litigation expenses such as the costs of legal counsel and of discovery. Indirect costs include business risks of patent litigation. While litigation is ongoing, the patentee and alleged infringer may find it optimal to adjust investments in innovation and new product releases. For example, the alleged infringer may prefer not to release new products prior to completing litigation, out of fear of being sued for willful infringement. L_I also includes opportunity costs of committing personnel to assisting with litigation instead of doing other things for their respective firms. Because full-blown litigation takes longer than settlement prior to litigation, it will tend to consume more resources. Savings on litigation costs, $2L(1-\omega)$, represents the remainder of the bargaining surplus.

By reaching a bargain with terms r and T, the parties avoid litigation and realize additional bargaining surplus

$$\begin{aligned}\Delta S = \; & U^P(\pi, T, r) - \left[\pi U^P(1, D, r) + (1-\pi)U^P(0,0,0)\right] \\ & + \left\{U^R(0, T, r) - \left[\pi U^R(1, D, r) + (1-\pi)U^R(0,0,0)\right]\right\} + 2(1-\omega)L.\end{aligned}$$

The terms on the first line give the expected additional total rents accruing to the patentee. The terms in brackets on the second line give the expected additional total rents accruing to the alleged infringer. The third term is savings on litigation costs. Under Nash bargaining, the patentee and alleged infringer maximize and then split this surplus. In equilibrium, the royalty equals r^*, the fixed payment equals T^*, and they each earn their expected payoff under litigation plus half of ΔS.

Conditional on an efficient bargain, this leads to the following payoffs:

$$\begin{aligned}V_S^P = \; & \pi\widehat{D} - \omega L + \tfrac{1}{2}\Big\{U^P(\pi, T^*, r^*) + \pi U^P(1, \widehat{D}, \widehat{r}) \\ & + (1-\pi)U^P(0,0,0) + U^R(0, T^*, r^*) - \Big[\pi U^R(1, \widehat{D}, \widehat{r}) + (1-\pi)U^R(0,0,0)\Big]\Big\} \\ V_S^R = \; & -\pi\widehat{D} - \omega L + \tfrac{1}{2}\Big\{U^R(0, T, r) + \pi U^R(1, \widehat{D}, \widehat{r}) \\ & + (1-\pi)U^R(0,0,0) + U^P(\pi, T^*, r^*) - \Big[\pi U^P(1, \widehat{D}, \widehat{r}) + (1-\pi)U^P(0,0,0)\Big]\Big\}\end{aligned}$$

Recall Figure 1. Start at point A and take expectations there. Then move to point B and take expectations there. Expected firm values change (from A to B) in the following ways:

$$
\begin{aligned}
\Delta V^P = \quad & (1-\lambda)\Big\{\pi\widehat{D} + \tfrac{2-\theta}{2}\Big[\pi U^P(1,\widehat{D},\widehat{r}) + (1-\pi)U^P(0,0,0)\Big] - U^P(\pi,0,0) \\
& +\tfrac{\theta}{2}\Big[U^R(0,T^*,r^*) + U^R(\pi,T^*,r^*) - (\pi U^R(1,\widehat{D},\widehat{r}) + (1-\pi)U^R(0,0,0))\Big] \\
& -L(\theta\omega + 1 - \theta)\} \\
\Delta V^R = \quad & (1-\lambda)\Big\{-\pi\widehat{D} + \tfrac{2-\theta}{2}\Big[\pi U^R(1,\widehat{D},\widehat{r}) + (1-\pi)U^R(0,0,0)\Big] - U^R(\pi,0,0) \\
& +\tfrac{\theta}{2}\Big[U^R(0,T^*,r^*) + U^P(\pi,T^*,r^*) - (\pi U^P(1,\widehat{D},\widehat{r}) + (1-\pi)U^P(0,0,0))\Big] \\
& -L(\theta\omega + 1 - \theta)\}
\end{aligned} \tag{1}
$$

These expressions highlight what the event study picks up.

1. Expected Damages: By construction, these are positive for the patentee and negative for the defendant:

$$
\begin{aligned}
P &: \pi\widehat{D} \\
R &: -\pi\widehat{D}.
\end{aligned}
$$

Although litigation is carried through to a decision with just probability $1-\theta$, the patentee earns expected damages in all scenarios—as a threat point under settlement, or as an expected damage payment in litigation. This is the only part of payoffs where the effect of litigation is unambiguously different for the patentee and defendant. It helps explain why we consistently estimate lower CARs for alleged infringers.

2. Change in Rents: Part of firm value changes because expected rents change. For the patentee and defendant, these are, respectively:

$$
\begin{aligned}
P &: \tfrac{2-\theta}{2}\Big[\pi U^P(1,\widehat{D},\widehat{r}) + (1-\pi)U^P(0,0,0)\Big] + \tfrac{\theta}{2}U^P(\pi,T^*,r^*) - U^P(\pi,0,0) \\
R &: \tfrac{2-\theta}{2}\Big[\pi U^R(1,\widehat{D},\widehat{r}) + (1-\pi)U^R(0,0,0)\Big] + \tfrac{\theta}{2}U^R(0,T^*,r^*) - U^R(\pi,0,0).
\end{aligned}
$$

Note that the nature of Nash bargaining payoffs dampens the effect that settlement has on rents. If $\theta = 0$, so that litigation is certain, then these payoffs are just expected rents under litigation minus status-quo rents under "no litigation." If $\theta = 1$, on the other hand, then settlement is certain and the patentee rents created through the bargain are shared with the defendant, so the patentee gets just one-half of those rents.

3. Extra Surpluses Achieved Through Bargaining: Part of firm value changes because the firm expects to earn some share of the change in rents from the other party. They are:

$$P : \tfrac{\theta}{2}\left\{U^R(0,T^*,r^*) - \left[\pi U^R(1,\widehat{D},\widehat{r}) + (1-\pi)U^R(0,0,0)\right]\right\}$$
$$R : \tfrac{\theta}{2}\left\{U^P(\pi,T^*,r^*) - \left[\pi U^P(1,\widehat{D},\widehat{r}) + (1-\pi)U^P(0,0,0)\right]\right\}.$$

Generally, we expect this change to be second-order.

4. Litigation Costs: Each party expects to spend resources to either settle litigation or carry it through. These are:

$$P : -L(\theta\omega + 1 - \theta)$$
$$R : -L(\theta\omega + 1 - \theta).$$

For both firms, value changes due to expected litigation costs. This will tend to be high either when settlement is unlikely (θ is low) or when settlement is costly to implement (ω is high).

In our empirical analysis, we find no effect of litigation on the firm values of patentees. Imposing $\Delta V^P = 0$ onto the first expression in (1), then substituting into the second expression, we identify the effect of litigation on the values of both firms in ΔV^R.

Proposition A1. *Suppose* $\Delta V^P = 0$. *Then*

$$\begin{aligned}\Delta V^R = \;& (1-\lambda)\{-2L(\theta\omega + 1 - \theta) \\ & +\theta U^P(\pi,T^*,r^*) + (1-\theta)\left[\pi U^P(1,\widehat{D},\widehat{r}) + (1-\pi)U^P(0,0,0)\right] - U^P(\pi,0,0) \\ & +\theta U^R(0,T^*,r^*) + (1-\theta)\left[\pi U^R(1,\widehat{D},\widehat{r}) + (1-\pi)U^R(0,0,0)\right] - U^R(\pi,0,0)\Big\}\end{aligned} \quad (2)$$

The first term inside brackets is the total litigation costs faced by *both* firms. The terms on the second line give the difference in expected rents for the patentee from "no litigation" versus the expected rents at the point litigation is initiated. The terms on the third line give the analogous difference for the expected rents for the alleged infringer.

Proposition A2. *Suppose* $\Delta V^P = 0$ *and that joint rents are not strictly maximal under the baseline. Then for sufficiently high* θ, *we have* $\Delta V^R > -(1-\lambda)2L(\theta\omega + 1 - \theta)$. *That is,* ΔV^R *understates expected litigation costs.*

Proof of Proposition A2. Because r and T maximize joint rents, these rents must exceed the joint status quo rents, i.e., $U^P(\pi, T^*, r^*) + U^R(\pi, T^*, r^*) - [U^P(\pi, 0, 0) + U^R(\pi, 0, 0)] > 0$. For $\theta = 1$, we have

$$\begin{aligned}\Delta V^R &= (1-\lambda)\{-2L\omega \\ &\quad + U^P(\pi, T^*, r^*) + U^R(\pi, T^*, r^*) - [U^P(\pi, 0, 0) + U^R(\pi, 0, 0)]\} \\ &> (1-\lambda)(-2L\omega).\end{aligned}$$

By continuity, $\Delta V^R > -(1-\lambda)2L(\theta\omega + 1 - \theta)$, for smaller values of θ. **QED**.

Intuitively, when litigation is filed, the values of the firms incorporate expected future litigation costs and the expected change in future rents. The change in rents must be positive as long as settlement is sufficiently likely, because settlement maximizes joint rents. For the special case where baseline rents happen to be jointly optimal, settlement merely reinforces baseline rents. Then, ΔV^R exactly equals $(1-\lambda)$ times litigation costs when $\theta = 1$. It may overstate $(1-\lambda)$ times costs when $\theta < 1$, because expected rents under litigation could be below the (jointly optimal) baseline. Then, ΔV^R will understate litigation costs provided the ex ante probability of litigation λ is sufficiently high.

Thus far, we have treated all litigation costs as being symmetric. However, it seems more realistic that (in the absence of a counter-suit) the patentee and alleged infringer will face asymmetric indirect costs. In particular, if the threat of willfulness claims drive the decisions that determine indirect costs, then the alleged infringer will face far higher indirect costs than the patentee. Suppose indirect costs remain L_I for the alleged infringer but are approximately zero for the alleged infringer. Then we have, for $\theta = 1$,

$$\begin{aligned}\Delta V^R = \ & (1-\lambda)\{-(2L_D + L_I)\omega \\ & + U^P(\pi, T^*, r^*) + U^R(\pi, T^*, r^*) - [U^P(\pi, 0, 0) + U^R(\pi, 0, 0)]\}\end{aligned}$$

For this case, the event study yields an upper-bound estimate for the combined direct litigation costs of both firms, plus the alleged infringer's indirect litigation costs.

2. Additional Estimates of Costs

Tables 10-11 include robustness checks on our estimates of average and aggregate litigation costs. We consider a different sample construction for some estimates. We use the full 25-day sample for others.

We construct the trimmed* samples to exclude observations where the event window overlaps with another event involving the same firm (as the trimmed samples from the paper do), and to exclude observations where the *estimation* window overlaps with another event involving the same firm. This sharply reduces the sample size but could (in principle) produce more efficient estimates because it eliminates all noise of other events during the estimation window. Table 10 is, for the trimmed* samples, analogous to Table 4 from the current version of the paper. Table 11, which reports average and aggregate cost estimates, is analogous to Table 5. This table also includes estimates of average and aggregate costs using the 25-day full sample (second column).

Table 10 shows that for the trimmed* samples, the average CARs for patentee events are not statistically different from zero. But the average CAR estimate for the 25-day trimmed* sample of alleged infringer events, -0.72%, is higher (in absolute value) than all other average CAR estimates in the paper. It is significantly higher (in absolute value) than the average CAR estimate for the 5-day trimmed* sample of alleged infringer events, -0.27%. Thus, the 5-day trimmed* sample significantly underestimates percentage losses and it would be unwise to base aggregate cost estimates on average event costs constructed from them.

In principle, the 25-day trimmed* sample could produce good estimates of average litigation costs. Indeed, we used this sample to estimate average and aggregate costs in a previous version of the paper. These estimates are reproduced in the first column of Table 11. Under the Average CAR Method—which is what we used previously—the average cost per event for alleged infringers using the 25-day trimmed* sample is $62.5 million. This is about 50% higher than the primary estimate of the average cost of $41.4 million per alleged infringer from the paper. Estimates are higher under the Individual CAR method and the Linear Model Method for the 25-day trimmed* sample.

Estimating aggregate costs using the 25-day trimmed* sample is awkward, because trimming removes about 60% of the sample. In a previous version of this paper, we estimated aggregate costs by assuming events outside the trimmed* sample had the same ($62.5 million) average cost as events in the trimmed* sample. This method yields an aggregate cost of about $538 billion over 1984-2009 (this appears in the Table under "Total Costs (N=8,607, scaled over all)"). This, too, is higher than the estimate of aggregate costs in the current version of the paper.

The second column of Table 2 estimates average and aggregate costs for the 25-day full sample, for which we report average CARs in Table 4. These results are comparable to related past work, such as Bessen and Meurer (2008; 2013b) and Bessen, Ford and Meurer(2011). Under the Average CAR Method (which these prior papers applied), the average cost using the full 25-day sample is $86.5 million. This is similar to estimates from these previous papers, and is also far higher than our primary $41.4 million estimate in the paper. Estimates are higher under the Individual CAR method and about the same under the Linear Model Method. Hence, all of these methods of estimating average and aggregate costs are not as conservative as the method used in the paper.

	Trimmed* Samples 5-Day Window	25-Day Window
All Event Parties		
N	5,709	5,357
Mean	-0.19%***	-0.51%***
	(0.06%)	(0.13%)
Median	-0.37%	-0.71%
Alleged Infringers		
N	3,664	3,458
Mean	-0.27%***	-0.72%***
	(0.07%)	(0.16%)
Median	-0.44%	-0.89%
Patentees		
N	2,045	1,899
Mean	-0.03%	-0.11%
	(0.10%)	(0.23%)
Median	-0.24%	-0.41%

Table 10: *Estimated Average Cumulative Abnormal Returns, 1984-2009*

Note: These statistics reflect an event study of patent litigation by all public firms matched to either a *Derwent* record or a *Patent Freedom* record during 1984-2009. We use equation (1) from the paper. The sample excludes all observations where the event window or the estimation window overlaps with the event window of another event involving the same firm. To estimate mean cumulative abnormal returns (CARs), we weight each individual CAR by the inverse of the variance of the estimated effect of the event. If two or more firms are involved in the same case, then each firm's participation is treated as a separate event. The "Patentees" category includes all firms clearly identified as patentees in known infringement suits or declaratory judgments, as well as plaintiffs in cases where we do not know if the case was an infringement suit or a declaratory judgment. The "Alleged Infringers" category includes all firms clearly identified as alleged infringers in known infringement suits or declaratory judgments, as well as defendants in cases where we do not know if the case was an infringement suit or a declaratory judgment.

	(1) 25-Day Trimmed* Sample	(2) 25-Day Full Sample
Average CAR		
N_E (Estimation)	3,458	8,607
Mean	62.5	86.8
Median	7.6	11.3
Total Costs (N_E)	216,140.7	746,891.4
Total Costs ($N = 8,607$, scaled over all)	537,976.6	746,891.4
Individual CAR Method		
N_E (Estimation)	3,458	8,607
Mean	131.1	273.8
Median	1.7	2.1
Total Costs ($N = N_E$)	453,283.4	2,356,799.0
Total Costs ($N = 8,607$, scaled over all)	1,128,227.4	2,356,799.0
Linear Model Method		
N_E (Estimation)	3,217	8,229
Mean	66.9	86.4
Median	4.6	5.2
Total Costs ($N = N_E$)	215,101.8	710,692.5
Total Costs ($N = 8,607$, scaled over all)	575,499.3	743,338.2

Table 11: *Estimates of Aggregate Costs, 1984-2009*

Note: All results use the CARs from event studies performed according to the note in Table 5, using just alleged infringer events from the 25-day trimmed* sample and the 25-day full sample. For each of the three methods used to estimate aggregate costs, we estimate a loss for each event and and then sum the losses. To estimate each event cost, the Average CAR Method multiplies the average CAR from Table 5 by the market value of the event party. In contrast, the Individual CAR Method multiplies the CAR for each event by the firm's market value. Finally, the Linear Model Method uses the model from (**??**) to predict CARs. Then, we multiply the predicted CAR by the event party's market value. All calculations are performed in STATA. The "scaled up to N=8,607" estimates calculate the average cost among all events in the trimmed sample. This cost is then imparted to all events not in the trimmed sample. Estimates are in $millions (2010)