Unbundling Ownership and Control*

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Abstract

Treating control as an asset that can be bought and sold, we introduce a model of the simultaneous and separable trading of ownership and control in a private information setting. The model provides a novel explanation for the prevalence and persistence of the separation of ownership from control in modern corporations: efficiency in the market for corporate control is more easily achieved when ownership is not concentrated in the hands of the manager. The central reason is that low managerial ownership reduces informational rents in the market for control. Using a mechanism design approach, we fully characterize the optimal mechanism for restructuring ownership and control. Under the optimal mechanism, corporations typically increase the number of shares of the incumbent manager if he remains in control, and give him a generous golden parachute that includes both stock and cash if he is deposed. By contrast, combining ownership and control is optimal only if agency costs are extreme.

Keywords: Ownership, corporate control, restructuring, mechanism design

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1 Introduction

We develop a model of the simultaneous and separable trading of firm ownership and control in a private information setting. We model corporate control as an asset that can be bought and sold, as first suggested by Manne (1965). That is, in our model firms can unbundle cash flow rights and control rights and sell them separately on the market. Starting from an initially exogenous ownership and control structure, our main problem is to choose how to divide a firm’s cash flow rights among its shareholders and how to allocate control; in other words, how to restructure control and ownership efficiently. We call the shareholder who is in control the manager. The (privately known) talent of the manager determines profits. Thus, efficiency requires that the most talented shareholder should be the manager.

There is separation of ownership from control whenever the manager holds less than 100% of the cash flow rights. We find that efficiency considerations in the market for control encourage the separation of ownership from control in corporations from both an ex ante and an ex post perspective. Ex ante separation facilitates efficient restructuring of ownership and control by increasing expected gains from trade for pivotal types of shareholders, making it less costly to induce participation in a restructuring mechanism. Ex post separation of ownership and control enhances the efficiency of the market for control by reducing informational rents. Unlike previous explanations, in our model the persistence of the separation between ownership and control relies on neither financing constraints nor risk aversion. Instead, it is driven by adverse selection problems and participation requirements. Agency costs emerge for sufficiently low managerial ownership and work in the opposite direction, preventing a complete separation between ownership and control.

The intuition is most easily seen by considering the extreme case of no agency costs. There, first-best restructuring is implemented with an efficient reassignment of managerial control only, in a game in which all shareholders truthfully report their types in equilibrium. In this “trivial” mechanism, shareholders benefit proportionately from any gains to changing managers, so informational rents (i.e., adverse selection problems) are eliminated and participation constraints are easily met. However, if the manager must own a sufficiently large share to preclude agency costs, then efficient restructuring requires trading of shares, creating informational rents that make participation constraints more difficult to meet. Ex post share rules (i.e., allocations of ownership shares to all shareholders, conditional on the assignment of control) that separate ownership and control economize on such rents. However, there may be no ex post share rule that implements the first-best as long as agency costs exist.

Our main results define necessary and sufficient conditions that characterize when a corporation can be efficiently restructured. We provide a full characterization of all feasible, incentive
compatible restructuring mechanisms, which are sets of rules to allocate control, shares, and cash transfers among market participants in an efficient way. The novel share rules for these mechanisms are related to, though distinct from, those in Vickrey-Clarke-Groves mechanisms. The optimal restructuring mechanism, which we show to be (virtually) unique, is the one that maximizes the net surplus from the restructuring activity. In other words, it implements an efficient allocation if and only if such an outcome is feasible.

The key to the optimal restructuring mechanism is the optimal share rule. The properties of this value-maximizing share rule have broad positive implications for the nature of corporate restructuring. Most importantly, this rule recommends separation of ownership from control in nearly every circumstance. The reason is that this separation loosens participation constraints in the market for corporate control by reducing informational rents. As a result, it enhances efficiency. The optimal share rule combines ownership and control only if agency costs are extreme.

The optimal share rule also highlights the asset value of control, first discussed by Manne (1965), as it rewards the ex ante manager with more shares than the ex ante non-controlling shareholders. The reason is that control makes the manager’s willing participation costlier, so he enjoys extra rents. Greater ex post shares lower such rents, making it “cheaper” to induce the manager’s participation. This is a novel rationalization of golden parachutes, as we show that there are strong efficiency reasons for including ownership shares (rather than cash transfers only) as part of a deposed manager’s compensation.\(^1\)

We use the optimal share rule further to characterize the set of ex ante ownership structures for which efficient restructuring is possible. We find that efficient restructuring is possible only if managerial ownership is sufficiently small. For higher levels of managerial ownership, management entrenchment effects preclude efficiency. Hence, ex ante separation of ownership from control also enhances efficiency. However, since lower levels of ex ante managerial ownership are more likely to generate agency costs, the goals of providing incentives to managers and facilitating control transfers conflict with each other. Our results suggest that the ideal level of managerial ownership is the lowest possible value such that there are no agency costs.

This last result leads to some unique empirical predictions. We find that the ownership structure of a corporation affects the frictions in the market for corporate control and, therefore, firm value. In their seminal contribution, Morck, Shleifer and Vishny (1988) find a non-monotonic empirical relationship between management ownership and firm value and, as an explanation, they offer an

\(^{1}\)Strictly speaking, we show that the optimal severance package for a departing manager must include compensation that is contingent on the value of the firm’s equity after the change in control. Golden parachutes that include equity-based pay are common in practice.
informal theory of the trade-off between managerial incentives and entrenchment. On the one hand, high management ownership is beneficial because it provides incentives to managers. On the other hand, they argue that excessive concentration of voting rights in the hands of management leads to inefficient entrenchment. Our model, however, generates entrenchment as a consequence of excessive ownership of cash flow rights rather than voting rights. Thus, unlike previous theories of entrenchment, our model predicts that firm value would eventually decrease as managers’ ownership of cash flow rights increases, even after keeping voting rights constant.

Recent evidence by Gompers, Ishii, and Metrick (2006) supports this prediction. Using a large sample of US firms that have dual class share structures, they are able to estimate the effect of company insiders’ ownership of cash flow rights on the market value of equity, while keeping the ownership of voting rights constant. They find an inverted U-shaped relationship: firm value first increases and then decreases with insiders’ ownership of cash flow rights. To our knowledge, ours is the first model that explains a negative effect of management ownership of cash flow rights on value.

We restrict attention to “linear” share ownership rules and to first-best mechanisms. Thus, our mechanisms do not include stock options or other non-linear share rules and we do not identify “second-best” mechanisms in cases in which efficient restructuring is impossible. Since stock options (and other non-linear instruments for contracting on profits) are essentially just more sophisticated techniques for separating ownership from control, their absence from our model actually strengthens our findings about the efficiency of unbundling. However, this absence does limit the ability of our model to explain existing practical mechanisms. Also, while we fully characterize the efficiency benchmark, we do not derive second-best mechanisms, which are inefficient but may still be Pareto-improving. The main consequence of this limitation is that the predictive power of our results is higher for friendly mechanisms for transferring control and lower for hostile mechanisms, where one or more shareholders does not willingly participate. Given that Boone and Mulherin (forthcoming) find that a significant percentage of acquisitions are indeed friendly, our results apply directly to an important set of cases.

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2 In contrast, Stulz (1988) provides a formal theory of the trade-off between higher takeover premia and the probability of takeover. For surveys of the empirical literature on the relationship between ownership structures and firm value, see for example Demsetz and Villalonga (2001) and Adams and Ferreira (2007).

3 As in many mechanism design applications, second-best mechanisms are technically intractible. Two notable exceptions are Myerson and Satterthwaite (1983), who show that the optimal second-best mechanism for bilateral exchange is a double auction, and Jehiel and Pauzner (2006), who identify second-best mechanisms for dissolving a particular class of partnerships.

4 A common mechanism for friendly acquisitions is a multiple-round auction, followed by negotiations, where a financial intermediary facilitates the bidding and subsequent contract construction.
1.1 Related literature

The literature on the separation of ownership from control is long and varied. In their seminal contribution, Berle and Means (1932) argue that separating control from ownership is detrimental for firm value because managers who are not owners will not be guided by profit-maximizing motives. Jensen and Meckling (1976) strengthen this vision by showing that the imperfect alignment of incentives between (controlling and owning) managers and (owning) shareholders fosters a value-reducing agency problem, which could nevertheless be mitigated if managers held stock.

Yet the separation of ownership from control is ubiquitous in the modern corporation. In both the U.S. and the U.K., many large firms have dispersed ownership structures in which managers can have real control over corporate resources despite owning a small number of shares. In continental Europe and many other countries, dispersed ownership is less common, but control rights and cash flow rights are still usually separated by a number of mechanisms, such as dual-class shares, pyramidal ownership structures, and the like.

There are numerous theories for the existence of this separation. Nearly all of them rely on either financing constraints or risk aversion, in contrast to our approach. In the setting of Jensen and Meckling (1976), for example, ownership and control are optimally combined unless entrepreneurial wealth is limited. When growth must be financed, there are agency costs to both debt and equity, and the optimal structure of the firm balances these costs at the margin, while introducing some separation of ownership from control. Demsetz (1983) argues that diffuse ownership may be an optimal response to an environment where a large scale of operations is essential to survival but where entrepreneurial wealth is limited.

The presence of risk aversion also presents a rationale for the separation of ownership from control. Fama (1980) and Fama and Jensen (1983) argue that risk-bearing and entrepreneurial decision-making are subject to specialization economies. Moreover, the principle of risk-sharing and limited liability both encourage diffuse ownership of shares.

Papers in the initial public offering (IPO) literature argue that entrepreneurs choose an optimal (from their standpoint) degree of separation of ownership from control when they decide to go public. This may be done either to provide appropriate incentives for monitoring management (Holmström and Tirole 1993, Pagano and Röell 1998) or to obtain a larger private value by first selling some ownership to atomistic shareholders (Zingales 1995). Our study focuses instead on how separating ownership from control economizes on endogenous transactions costs (such as informational rents) in the market for corporate control.

Our results also relate closely to the literature on possible failures of the market for corporate control, initiated with the seminal work of Grossman and Hart (1980). In their paper, because
of the free-riding behavior of small shareholders, “too few” changes in control occur. Most of the subsequent literature on tender offers (e.g., Shleifer and Vishny 1986, Burkart 1995, Burkart et al. 1998, Singh 1998, Bulow, Huang and Klemperer 1999, Burkart et al. 2006) suggests that the ownership structure of a firm has effects on the functioning of market mechanisms to restructure ownership and control (for example, through toeholds). The results in that literature, while insightful, are vulnerable to the critique of Demsetz (1983): if a specific takeover mechanism does not lead to an efficient outcome, why not use a different one? For example, Grossman and Hart (1980) show that if dilution of original shareholders is possible, the free-riding problem is eliminated. Although dilution is illegal in the U.S., Mueller and Panunzi (2004) show that the same outcome can be achieved when the raider finances its acquisition by issuing debt backed by the target’s future cash-flows. They argue that these “bootstrap acquisitions” are legal and were widely used in the takeover wave of the 1980s. Our approach, by contrast, is not subject to Demsetz’s critique. In appealing to the revelation principle, we permit the use of any restructuring mechanism. Thus, our finding that ownership is value-relevant is robust to all available mechanisms.

The theoretical underpinnings of our mechanism design approach relate to Cramton, Gibbons and Klemperer (1987), the first paper to study efficient dissolution of partnerships in the presence of asymmetric information. Our model departs from their framework in four important ways: (1) shareholder valuations are interdependent; (2) control is tradable; (3) ownership and control are separable ex ante and ex post; and (4) agency costs may exist for sufficiently low managerial ownership. As in the model of Ornelas and Turner (forthcoming), the manager’s ability alone determines the profit of the firm, so valuations of ownership shares are interdependent. As Fieseler et al. (2003) and Ornelas and Turner show, interdependence significantly affects the possibility of efficient implementation. In particular, the Cramton et al. (1987) finding that equal-shares partnerships can always be efficiently dissolved does not extend to the case of interdependent valuations.

More importantly, when ex post control and ownership are separable, as in our model, allocative efficiency depends both on the assignment of control and on a minimum level of ex post managerial ownership. This greatly expands the set of share rules (and mechanisms) capable of potentially implementing the first-best, as efficiency no longer requires reducing the firm to single ownership. These results help explain why partnerships with control structures like the one in our model might prefer to remain intact rather than dissolve.

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5 Burkart, Gromb and Panunzi (2000) and Bebchuk and Hart (2001) provide comparative analyses of two different mechanisms.

6 That paper led to extensive work on dissolving partnerships—see for example McAfee (1992), Moldovanu (2002), Fieseler, Kittsteiner and Moldovanu (2003), Jehiel and Pauzner (2006) and Ornelas and Turner (forthcoming).
This paper contributes also to the broader mechanism design literature. Since buyer/seller exchange is a special type of restructuring, and therefore a special case of our model, we are able to generalize the analysis of Myerson and Satterthwaite (1983) for the pure exchange of ownership to the simultaneous and separable exchange of control and ownership. We show that efficient bilateral exchange of control alone is actually possible for identical, continuous types of buyer and seller. However, this is possible only if the full separation of ownership and control introduces no agency costs.

2 The Model

An all-equity firm is initially owned by n risk-neutral shareholders indexed by $i \in N \equiv \{1, ..., n\}$. Shareholder $i$ owns a fraction $r_i \in [0, 1]$ of shares, and $\sum_{i=1}^{n} r_i = 1$. There are no wealth constraints. Ownership does not imply control over the decisions taken within the firm. Instead, the firm resembles a modern corporation, in that a team of professional managers is in charge of running it. To abstract from conflicts of interest within the management team, we model this team as a single individual with full control. We refer to the initial manager as shareholder 1. Thus, $r_1$ is our measure of managerial ownership.

We denote the general ability of shareholder $i$ in running the firm by $a_i$.\(^7\) We assume that $a_i$ is distributed according to an increasing, continuous and differentiable cumulative distribution function $F$ with support $[a, \bar{a}]$. Managerial talent is private information and is not ex post verifiable. Thus, shareholder $i$ knows his own ability $a_i$, but any shareholder $j \neq i$ knows only the distribution of $a_i$. The expected value of $a_i$ is denoted by $\mu$.

Our economy includes all potential managers for the firm. This may include individuals who initially do not own shares, so it is possible that $r_i = 0$ for one or more $i$. We consider a simple technology in which expected profit $\pi$, conditional on knowledge of the manager’s ability, is a linear function of that ability. Thus, under the initial control structure and in the absence of agency costs, the manager knows that expected profit will be $\pi = a_1$, whereas the non-controlling shareholders expect profit $E\{a_1\} = \mu$. If upon restructuring shareholder $i$ becomes the manager, the firm’s expected profit becomes $\pi = a_i$.\(^8\)

Managers may have incentives to divert company profits to their private gain. To model this in the simplest possible way, we assume that there is a threshold managerial ownership share, $s \geq 0$, above which the manager is discouraged from diverting profits. A high $s$ represents a situation

\(^7\)We treat $a_i$ as a measure of managerial talent, but other interpretations are also possible. For example, $a_i$ might be considered a measure of shareholder $i$’s ability to identify the right people who will actually run the business.

\(^8\)When there is no confusion, we drop the "expected" modifier to profit.
where agency costs are important and therefore require a high level of managerial ownership to prevent diversion of profits. A low $s$ reflects the opposite situation, where agency costs are of lesser relevance. Because we model agency costs in a simplified, reduced-form way, we abstract from the effects of those costs on firm profits. That is, we motivate the efficiency requirement that the ex post managerial share must be at least $s$ by appealing to these unmodeled agency costs. In line with that assumption, we restrict attention to the case where $r_1 \geq s$ so that the initial ownership share prevents agency costs.

The timing of events is as follows. There is an initial, exogenous allocation of control and of ownership, $r = \{r_1, r_2, ..., r_n\}$. After ownership and control are allocated, each shareholder learns his ability. They then write a multilateral contract to reallocate ownership and control among themselves. Under the rules of this contract, they implement a new allocation of shares and control rights. Finally, production takes place and the firm generates expected profit $\pi = a_j$, where $j$ is the index of the (potentially) new manager in charge. We refer to this sequence of events as the operation of a restructuring mechanism, which is a procedure to change the original structure of ownership and control. We refer to the set of available restructuring mechanisms as constituting the market for control.

2.1 The restructuring problem

Suppose that there were no scope for agency costs ($s = 0$) and no private information. It is then obvious that, without direct costs of restructuring, the first-best efficient allocation could always be achieved, with control being assigned to the most talented shareholder regardless of the initial ownership and control structures. This is, in fact, a simple illustration of the Coase Theorem. Since our model imposes very few restrictions on how shareholders can restructure ownership and control, it is not surprising that ex ante ownership is irrelevant and efficient restructuring can always be achieved in our setup in the absence of direct transaction costs. The expected surplus from restructuring in this case is the first best, $V^{fb} \equiv \mathbb{E}(\bar{a} - a_1)$, where $\bar{a} \equiv \max\{a_1, ..., a_n\}$. The surplus from restructuring under asymmetric information is thus necessarily no higher than $V^{fb}$.

However, even when shareholders’ talent is private information, fully-efficient restructuring remains possible. We illustrate this possibility using a particularly simple mechanism.

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9It is possible to incorporate such costs fully in the model without any change in the qualitative results, however. For example, a previous version of this paper (Ferreira et al. 2005) adopts a slightly modified version of Burkart et al.’s (1998) modeling of extraction of private benefits by managers and yields an endogenous $s$ and endogenous agency costs. Alternatively, one could follow the modeling approach of Shleifer and Wolfenzon (2002) and obtain similar results.

10We adopt the convention that **bold** variables represent vectors.
Definition 1 The trivial restructuring mechanism has the following characteristics. After learning his ability \( a_i \), each shareholder simultaneously announces his type. The mechanism assigns control to the agent who reports the highest ability \( \bar{a} \), while the ownership structure \( r \) remains intact throughout.

It is immediate to see that the trivial mechanism yields a Bayesian-Nash equilibrium, where all shareholders truthfully report their abilities.\(^{11}\) This mechanism implements the first-best allocation of control, the participation constraints of all shareholders are met, and the mechanism has a balanced budget. Thus, adverse selection \textit{per se} is not a problem for efficient restructuring, as long as contracts are complete. The reason is that, because all shareholders keep their shares, they benefit proportionally from the gains from trade, and there are no informational rents. In a sense, the trivial mechanism implements the direct opposite of a complete dissolution mechanism (such as the one in Cramton et al. 1987). Since dissolution bundles ownership and control, it introduces significant informational rents because all shareholders must surrender their shares when opting for the mechanism. This previews our main finding that separating ex post ownership from control enhances efficiency by reducing informational rents.

Implementing an efficient allocation of ownership and control poses difficulties only when both private information and agency costs are present. In that case, since the manager has incentives to divert company profits unless his ownership share is sufficiently large, the problem of value maximization requires not only assigning the right person to management, but also making sure that this person’s equity stake is large enough to prevent him from diverting profits. The trivial mechanism can then be expected to yield \( V^{fb} \) only if the initial ownership share of each shareholder is sufficiently high to preclude each shareholder from diverting profits if he happens to become the new manager. In the typical case where initial ownership shares do not satisfy that requirement, ownership will need to be reassigned in an efficient (non-trivial) restructuring mechanism to guarantee a sufficiently high manager’s share.

2.2 Mechanisms for efficient allocation of ownership and control

A corporation \( (r, F) \) is fully characterized by its ex ante ownership structure \( r \) and by the distribution of managerial abilities \( F \). Using the revelation principle, we study a direct revelation mechanism in which shareholders simultaneously report their types \( a = \{a_1, ..., a_n\} \) and the mechanism determines (1) the new control structure \( c(a) = \{c_1, ..., c_n\} \); (2) the new ownership structure

\(^{11}\) Note that this does not yield a Groves equilibrium. If all shareholders other than \( i \) announce ability \( a_i \), then shareholder \( i \) will strictly prefer to name \( \bar{a} \) if he thinks his type is worse than average. Generally, dominant strategy implementation is not possible with interdependent types, because of off-equilibrium-path scenarios such as this one.
s (a) = \{s_1, ..., s_n \}; and (3) transfer payments to shareholders t (a) = \{t_1, ..., t_n \}. We assume that
c_i \in \{0, 1\}, where c_i = 1 implies that shareholder i has control (so that π = a_i) and c_i = 0 implies
that he does not have control.

We restrict attention to mechanisms that are (ex ante) budget balanced. This requires

\[
\begin{align*}
\sum_{i=1}^{n} c_i (a) &= 1 \\
\sum_{i=1}^{n} s_i (a) &= 1 \\
E \left( \sum_{i=1}^{n} t_i (a) \right) &= -D,
\end{align*}
\]

where D \geq 0 is an exogenous constant that represents the direct cost of restructuring that must
be borne by the shareholders. This might consist of costs arising from regulations, trading costs,
raising funds to place a takeover bid, etc. In a world with no exogenous transaction costs, D = 0.
We call (c, s, t) a restructuring mechanism.

A necessary condition for a mechanism to be ex post efficient is that it allocates control according
to\[c_i = \begin{cases} 
1 & \text{if } a_i = \tilde{a} \\
0 & \text{if } a_i < \tilde{a}.
\end{cases}\]

An efficient mechanism must, additionally, preclude agency costs ex post. Letting s^{c_i}_i be the
ownership share of partner i conditional on his control c_i, this requires

\[
\begin{align*}
s^1_i &\geq \underline{a}.
\end{align*}
\]

Thus, while efficiency alone does not impose any constraint on the shares s^0_i received by any
shareholder i who does not assume control, it does require that shareholder i’s controlling share, s^1_i, must exceed \underline{a}.\[\text{14}\]

We allow s^0_i to be idiosyncratic across shareholders but assume that s^0_i is independent of the
identity of shareholder j \neq i who is assigned control. This assumption greatly simplifies the analysis
and is without loss of generality in the current setting, where all shareholders share the same ability
distribution F and are therefore identical from an ex ante perspective.

\[\text{12}\text{Clearly, the "ex ante" applies only to the transfers. When ex ante budget balance is satisfied, it is straightforward}
\text{to apply the techniques of d’Aspremont and Gérard-Varet (1979) to find ex post budget-balancing transfers.}\]

\[\text{13}\text{The case where two shareholders tie for highest type is a zero probability event and can be ignored.}\]

\[\text{14}\text{It is possible that other, exogenous forces, may require a minimum managerial ownership share as well. For}
\text{instance, } \underline{a} \text{ could be affected by legal or institutional forces that govern the required minimum share necessary for}
\text{acquiring control. For example, if a corporation is required to dissolve, then } \underline{a} = 1.\]
Budget balance and (3) impose the following restrictions on $s$. This and the subsequent proofs are in the Appendix unless they are very short.

**Lemma 1** In any efficient restructuring mechanism, budget balance implies:

1. \[ \sum_{i\neq 1} s_i^1 = (n - 1) \left( 1 - s_1^0 \right) - (n - 2) \left( 1 - s_1^1 \right) \geq (n - 1) s \] and

2. \[ \sum_{i\neq 1} \left( s_i^1 - s_i^0 \right) = (n - 1) \left( s_1^1 - s_1^0 \right). \]

### 2.3 Participation

Unanimous participation, where any single shareholder can block restructuring, is clearly a necessary condition for efficient implementation. If any shareholder who is potentially capable of being the manager chooses not to participate in the restructuring mechanism, then there is a positive probability that an inferior manager will assume control ex post.

At first glance, it may seem unrealistic to assume that all shareholders have blocking power. However, our assumption that all shares are owned by potential managers is just a normalization made for analytical simplicity. Our results do not change qualitatively if some portion of ownership exogenously belongs to atomistic shareholders, who do not have veto power because their shares remain outside the scope of the restructuring mechanism. Thus, it is appropriate to think of our shareholders as consisting of all candidates for future management. In many cases, this may consist of only a handful of important blocs of shareholders, so our results can be particularly important for “small $n$.”

In a more literal interpretation of our model, one can think of a closely-held corporation in which all its few shareholders sit at the negotiation table when considering whether to undertake a major change. Our model is not restricted to this case, but it is convenient to keep this interpretation throughout. Although we do not explicitly model the case with a fraction of dispersed, non-pivotal shareholders, such an extension is straightforward.

Veto rights to all shareholders are actually very common under "shareholder agreements" contracts signed by all shareholders of a firm specifying the rights, duties and rules of the game whenever there is a major change in the firm (normally, changes in ownership and control). They are heavily used when one of the large shareholders is also involved in management (such as joint

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15 For example, if an outsider wished to gain control of the Italian corporation Fiat, he would basically just need to deal with the Agnelli family, in which case there would be only two important groups of shareholders. Shares not owned by the outsider or by the Agnellis would be outside the scope of the restructuring mechanism, and those shareholders’ participation would not matter. As long as none of those other shareholders make viable candidates for management, there is no efficiency loss.
ventures and venture-capital-backed firms). Most importantly, they usually assign veto rights to each shareholder in case of major restructurings of ownership and control.\textsuperscript{16}

An admitted weakness of our approach is that when achieving the first-best is impossible, the unanimity rule is inefficient, because it may block Pareto-improving changes that lead to second-best outcomes. The presence of hostile takeovers in practice suggests that second-best mechanisms are, in some instances, the best to be hoped for. However, empirical research has shown that, despite the attention given to takeovers in the 1980s, hostile mechanisms are relatively infrequent.\textsuperscript{17} This suggests that, in addition to being analytically more manageable, the efficiency benchmark is empirically more relevant.

2.4 Incentive compatibility and individual rationality

Let $-i \equiv N \setminus i$ and $a_{-i} \equiv \{a_1, ..., a_{i-1}, a_{i+1}, ..., a_n\}$, and let $E_{-i} \{\cdot\}$ denote the expectation operator with respect to $a_{-i}$. Under mechanism $\langle c, s, t \rangle$, shareholder $i$ expects to receive transfer $T_i(a_i) \equiv E_{-i}\{t_i(a)\}$. He also expects to be allocated control with some probability and expects to own some shares ex post. Let $G \equiv F^{n-1}$ be the distribution of the largest of the other shareholders’ abilities, $\tilde{a}_{-i} \equiv \max\{a_1, ..., a_{i-1}, a_{i+1}, ..., a_n\}$, with corresponding density $g$. Thus, shareholder $i$’s interim expected utility under an ex post efficient mechanism $\langle c, s, t \rangle$ is

$$U_{im}(a_i, b) = s_1^i a_i G(a_i) + s_0^i \int_{a_i}^{\tilde{a}_i} udG(u) + T_i(a_i),$$

(4)

where the first argument of $U_{im}(\cdot, \cdot)$ is shareholder $i$’s ability and the second is his announced ability. In contrast, if no mechanism were put into place, the initial ownership and control structure would be kept intact and the shareholders would expect to receive the following dividends:

$$\left\{ \begin{array}{l}
U_1(a_1) = r_1 a_1 \\
U_i(a_i) = r_i \mu 
\end{array} \right. \text{ for all } i \in \{2, ..., n\}.$$  

(5)

We define the interim expected net utility from participating in the mechanism as $U_i(a_i, b) \equiv U_{im}(a_i, b) - U_i(a_i)$.

We specify the transfers in our mechanism as

$$t_i(a) = \left\{ \begin{array}{ll}
-k_i \\ -(s_i^1 - s_i^0) \tilde{a} - k_i
\end{array} \right. \text{ if } s_i(a_i) = s_i^1, \text{ if } s_i(a_i) = s_i^0,$$

(6)

\textsuperscript{16}Chemla et al. (2007, p. 117) describe some of the most common clauses in these agreements: “Provision of control. Designation of the rights and duties of the shareholders in the management of the company, and requirement of prior unanimous consent for major decisions such as the declaration of any dividend and the issuance or sale of shares. Restrictions on the transfer of shares. The shareholders commit not to sell, pledge, or charge their shares except with the prior written consent of all other shareholders.”

\textsuperscript{17}In the dataset used by Hartzell, Ofek and Yermack (2004), only 3% of the acquisitions were unsolicited.
where $k_i$ is a real number. These transfers have the nice feature of being affected by the announcement of each shareholder in a direct mechanism only through its effect on the allocation of control. In this sense, they are similar to the transfers from a standard Vickrey-Clarke-Groves mechanism, though the presence of $(s_i^1 - s_i^0)$ means they do not fit in that family. Note that the strength of the incentives in the transfers is largest under complete dissolution (i.e. in a Cramton et al. 1987-type of mechanism), and smallest under the trivial mechanism. Powerful incentives must be given to particular types to achieve truth-telling under complete dissolution, leading to high informational rents for those types. These incentives are not necessary under the trivial mechanism, as shares are kept constant.

The transfers defined in (6) imply that the mechanism is incentive compatible. To see this, note that conditional on all other shareholders declaring their types truthfully to the mechanism, shareholder $i \in N$ expects to receive a transfer of $T_i(b) = (s_i^1 - s_i^0) \int_{b}^{\bar{a}} udG(u) - k_i$ by announcing his ability as $b$ instead of its true value $a_i$. In that case, the net utility he achieves with the mechanism is

$$U_i(a_i, b) = s_i^1 a_i G(b) + s_i^0 \int_{b}^{\bar{a}} udG(u) + T_i(b) - \mathcal{T}_i(a_i) = s_i^1 a_i G(b) + s_i^1 \int_{b}^{\bar{a}} udG(u) - k_i - \mathcal{T}_i(a_i).$$

Since $dU_i(b)/db = s_i^1 g(b)[a_i - b]$, it follows that $U_i(a_i, b)$ is maximized when $b = a_i$ (it is straightforward to check that the second-order condition is satisfied at this point), confirming that the mechanism is incentive compatible. Intuitively, the transfer $t_i(a)$ is set to counteract the incentive that shareholder $i$ has to misrepresent himself as having a higher (lower) type when he expects to receive more (fewer) shares if he gains control than if he does not, i.e. if $s_i^1 > s_i^0$ (resp., $s_i^1 < s_i^0$).

With truth-telling ensured, the expected net utilities under the proposed mechanism can then be expressed as follows:

$$
\begin{cases}
U_1(a_1, a_1) = s_1^1 a_1 G(a_1) + s_1^1 \int_{a_1}^{\bar{a}} udG(u) - k_1 - r_1 a_1 \quad \text{and} \\
U_i(a_i, a_i) = s_i^1 a_i G(a_i) + s_i^1 \int_{a_i}^{\bar{a}} udG(u) - k_i - r_i a_i \quad \text{for all } i \in \{2, ..., n\}.
\end{cases}
$$

Fieseler et al. (2003) show that, with interdependent types, the interim expected net utility of each agent under a mechanism that is both efficient and incentive compatible is determined up to a constant. Thus, the transfers defined in (6) are the only ones that are both incentive compatible.

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18 The techniques of d’Aspremont and Gérard-Varet (1979), applied to the choice of $\{k_i\}$, lead to an ex post budget balancing mechanism. See Williams (1999).

19 Notice also the contrast with the literature on dissolving partnerships, where the assumption that $s_i^1 = 1$ and $s_i^0 = 0$ implies that each shareholder would always have an incentive to announce $b = \bar{a}$ in the absence of transfers.

20 See the proof of Theorem 1 of Fieseler et al. (2003), which follows arguments developed by Williams (1999).
and ex post efficient. Any efficient direct revelation restructuring mechanism must imply cash transfers as in (6).

Because incentive compatibility is ensured, one can always implement a direct mechanism when transfers are defined as in (6) with the help of external subsidies to guarantee participation. However, if there are no external subsidies, participation can only be guaranteed if the expected gains from restructuring are sufficiently large relative to the informational rents required to induce truth-telling. In reality, external subsidies are unlikely to be available. On the contrary, the implementation of a mechanism is likely to generate additional administrative costs, captured here by the constant $D$.

The worst-off type of shareholder $i$ in $(c, s, t)$ has the minimum net utility among all types:

$$a^*_i \in \arg \min_{a_i \in [a, \bar{a}]} \{U_i(a_i, a_i)\}.$$ (9)

We call a mechanism interim individually rational if all types of all shareholders have a non-negative expected net utility from the mechanism:

$$U_i(a^*_i, a^*_i) \geq 0 \text{ for all } i.$$ (10)

The requirement is equivalent to assuming that each shareholder $i$ can block transfers of ownership and control. The next two lemmas characterize the participation constraints of our mechanism.

**Lemma 2** An efficient mechanism $(c, s, t)$ with transfers defined as in (6) is individually rational for shareholder 1 if and only if

$$k_1 \leq \bar{s}_1 \int_{a^*_1}^{\bar{a}} udG(u) - \max\{(r_1 - s^1_1) \bar{a}, 0\},$$ (11)

where $a^*_1 = G^{-1} \left( \min \left\{ \frac{c}{s^1_1}, 1 \right\} \right)$ except when $s^1_1 = r_1 = 0$, in which case $a^*_1$ is any element in $[a, \bar{a}]$.

Lemma 2 identifies the worst-off type of manager and characterizes individual rationality for him. The worst-off type is increasing in $r_1$, as higher initial ownership means that there is more trading profit to be earned by being a seller, which a very high type does not wish to be. By contrast, the worst-off type of the manager is decreasing in $s^1_1$, as more ex post ownership means that the manager sells fewer shares (or, rather, buys more shares).

The next lemma characterizes the individual rationality constraints for the non-controlling shareholders.
Lemma 3 An efficient mechanism \((c, s, t)\) with transfers defined as in (6) is individually rational for shareholder \(i \in \{2, \ldots, n\}\) if and only if

\[ k_i \leq s_i^1 \int_{a_i^*}^{\bar{a}} udG(u) - r_i \mu, \tag{12} \]

where \(a_i^* = a\) unless \(s_i^1 = 0\), in which case \(a_i^*\) is any element in \([a, \bar{a}]\).

The intuition behind this result is simple. The worst-off type of a non-controlling shareholder has the lowest possible managerial ability \(a\).\(^{21}\) Such a shareholder knows that, leaving aside the fee \(k_i\) he would have to pay, his expected payoff under the mechanism is \(s_i^1 \int_{a_i^*}^{\bar{a}} udG(u)\), while he expects to receive \(r_i \mu\) if he does not participate. Thus, he participates only if the fee \(k_i\) is not larger than the difference between these two values.

Lemma 3 illustrates an effect that is similar to the free-riding behavior of non-controlling shareholders analyzed by Grossman and Hart (1980): non-controlling shareholders, who do not contribute for production, will hold on to their shares unless they are paid a premium over their current value. In Grossman and Hart’s specific bidding game, the price paid per share had to be at least equal to the share price after the change in control. Because we focus on the set of all implementable efficient mechanisms, we find that free-riding by non-controlling shareholders can be mitigated by means of considerably smaller price premia. In fact, due to the unanimity rule, this premium is zero. But even when non-controlling shareholders receive no rents from selling their shares, they will block efficiency-enhancing changes of control unless they get at least the value of their shares under the status quo. Thus, participation remains a problem even when these shareholders are pivotal.

3 Conditions for Efficient Restructuring

3.1 Exogenous ex post share rules

For ease of exposition, we begin the analysis of the feasibility of efficient restructuring by taking the ex post ownership structure \(s\), or share rule, as given. We derive the optimal share rule in the next subsection.

Expressions (2), (3), (6), (11) and (12) allow us to characterize the set of all ex ante and ex post structures for which an efficient restructuring mechanism achieves budget balance:

\(^{21}\)In the text, we ignore the uninteresting multiplicity of worst-off types arising in the boundary cases of lemmas 2 and 3.
Proposition 1 A corporation \((r,F)\) can be efficiently restructured with an incentive compatible, individually rational mechanism with share rule \(s\) and transfers defined in (6), and which satisfies ex ante budget balance, if and only if \(s\) satisfies lemma 1 and
\[
V(r,s) \geq D,
\]
where
\[
V(r,s) = \sum_{i=1}^{n} \left[ s_i^{1} \int_{a_i^*}^{a} uG(u) - (s_i^1 - s_i^0) \int_{a_i}^{a} F(u) udG(u) \right] - \max\{ (r_1 - s_i^1) \bar{a}, 0 \} - (1 - r_1)\mu
\]
and the \(\{a_i^*\}\) are as defined in lemmas 2 and 3.

Condition (13) compares the net surplus \(V(r,s)\), namely the expected gains from trade minus the informational rents generated by restructuring mechanism \((c,s,t)\), to its operating costs \(D \geq 0\). Whenever condition (13) holds, any “wrong” initial allocation of control can be efficiently corrected by a mechanism with share rule \(s\). On the other hand, if condition (13) is not met, then mechanism \((c,s,t)\) cannot achieve ex post efficiency unless there is an outside subsidy.

It is clear from Proposition 1 that the possibility of efficient restructuring does not depend on the initial distribution of ownership among non-controlling shareholders. That distribution affects whether the trivial mechanism implements efficient restructuring, as discussed earlier, but not \(V(r,s)\). In light of this, from now on we replace \(r\) by \(r_1\) in the argument of \(V\).

3.2 Optimal mechanisms

Proposition 1 takes the share rule as exogenous. However, if shareholders can bargain together and implement their decisions effectively, they will choose \(s\) optimally. In particular, they will limit themselves to the set of share rules capable of achieving \(V^{fb}\) whenever efficient restructuring is possible. In this subsection, we give necessary and sufficient conditions for whether this set is non-empty, that is, for whether corporation \((r,F)\) can be efficiently restructured. The key instrument is a particular share rule, defined next, which is in this set if and only if it is non-empty.

\footnote{The initial distribution of ownership among non-controlling shareholders does not affect the possibility of efficient restructuring because we abstract from outside shareholder monitoring.}
Definition 2  The optimal share rule $s(r_1)$ satisfies

$$s(r_1) \in \arg\max_{s \in B} V(r_1, s),$$

where $B$ is the set of all efficient share rules that satisfy budget balance. An optimal restructuring mechanism is an incentive compatible, individually rational, ex post efficient mechanism with ex post share rule $s(r_1)$.

Note that the continuity of the value function, $V(r_1, s(r_1))$, follows from the theorem of the maximum, while the existence of optimal restructuring mechanisms follows from the continuity of $V(r_1, s)$ with respect to $s$ and from the fact that $B$ is a non-empty compact set. The next proposition describes the pivotal nature of the optimal share rule and shows how it unbundles ownership and control.

**Proposition 2**  A corporation $(r, F)$ can be efficiently restructured if and only if $V(r_1, s(r_1)) \geq D$, where the optimal share rule $s(r_1)$ is unique and requires:

i. $s_i^0(r_1) = \frac{1-s_i^1(r_1)}{n-1}$ for all $i \neq 1$.

ii. $s_i^1(r_1) = \frac{1}{n} - \frac{n-2}{n-1} \left[ 1 - s_i^1(r_1) \right].$

iii. $s_0^0(r_1) = 1 - \frac{1}{n} - \frac{n-2}{n-1} \left[ 1 - s_1^1(r_1) \right].$

iv. $s_i^1(r_1) =\begin{cases} 0 & \text{if and only if } r_1 = 0 \\ 1 & \text{if and only if } r_1 > 0 \end{cases}$

Otherwise, $s_i^1(r_1) \in \left[ s_i^0, 1 \right]$ is interior and satisfies

$$\int_{a_w}^{\bar{a}} G(u)du = \bar{a} - \int_{a_w}^{\bar{a}} udF(u)^n, \quad (15)$$

where $a_w \equiv G^{-1}\left( \frac{r_1}{s_i^1(r_1)} \right)$. In any case, $s_i^1(r_1) \geq r_1$, with $s_i^1(r_1) > r_1$ for $r_1 \in (0, 1)$.

Intuitively, a mechanism using the optimal share rule achieves efficient restructuring for the largest possible direct cost $D$, so it is in the set of implementable share rules if and only if ex post efficiency is possible.\footnote{While the optimal share rule, $s(r_1)$, is unique, it is clear from (6) that there may be a multiplicity of sets of fixed side payments $\{k_i\}_{i=1}^n$ that yield $V(r_1, s(r_1))$.} In nearly every circumstance, such a mechanism does not combine
ownership and control ex post. Thus, Proposition 2 strongly suggests that an efficient market for corporate control reinforces the separation of ownership from control.

To see that more clearly, consider separately the manager and the non-controlling shareholders. If the manager retains control, the optimal share rule assigns full ex post ownership to him only if agency costs are extreme \( (\bar{s} = 1) \) or if his initial ownership \( r_1 \) is sufficiently high.\(^{24}\) On the other hand, if a non-controlling shareholder assumes control, he invariably receives an ownership share of \( s \). This is less than 1 for all non-extreme levels of agency costs. The following corollary summarizes these points.

**Corollary 1** The optimal share rule combines ownership and control with certainty \((s_1^1 = 1 \text{ and } s_i^0 = 0 \text{ for all } i)\) if and only if agency costs are extreme, i.e. \( \bar{s} = 1 \).

**Proof.** We show in the proof of Proposition 2 that \( V(r_1, s) \) is increasing in \( s_1^0 \). Therefore, unless \( \bar{s} = 1 \), the optimal \( s_1^0 \) is strictly positive, and \( s_1^1 < 1 \) for at least one \( i = 1, \ldots, n \).

Separating ownership and control is clearly inefficient when agency costs are extreme, so the optimal share rule must combine them in such a case. It is striking, however, that bundling ownership and control completely is sub-optimal in every other circumstance.

The reason is that the market for control is subject to less friction when shareholders expect the separation of ownership and control to persist. Intuitively, an increase in any "losing" share \( s_i^0 \) relaxes the participation constraint for shareholder \( i \) by reducing expected informational rents, because this share rule allocates some of the gains from trade to those shareholders without the largest ability (see equation 14). For that reason, losing shares are optimally set at strictly positive levels unless \( \bar{s} = 1 \).

Proposition 2 also makes clear that ex ante control has value, which emerges endogenously in our model. The optimal share rule treats the ex ante manager and the ex ante non-controlling shareholders quite differently, generally allocating more shares to the former \((s_1^1 \geq s_i^1 \text{ and } s_1^0 \geq s_i^0)\). The reason is that facilitating the manager’s participation is fundamentally distinct from facilitating the participation of others, since control yields an informational advantage to the manager because the status quo profit of the corporation is his private information. While the worst-off type of non-controlling shareholders, \( \underline{s} \), is unaffected by the share rule, the worst-off type of manager is lower when \( s_1^1 \) and \( s_1^0 \) are higher. As we discuss next, since facilitating participation by higher ability managers is more costly, the optimal share rule sets the ability of the worst-off type of ex ante manager at a relatively low level, accomplishing this by raising the manager’s ex post shares above those for ex ante non-controlling shareholders.

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\(^{24}\)To see when the second case holds, notice that \( \alpha - \int_{G^{-1}(r_1)}^{\bar{s}} G(u)du - \int_{\underline{s}}^{\bar{s}} u dF(u)^n > 0 \) is increasing in \( r_1 \).
The easiest way to see this is to note that increases in \( s_1^0 \) are more effective at reducing participation frictions than increases in the other \( s_i^0 \) shares. Budget balance implies that an increase in \( \sum_{i\neq 1} s_i^0 \) lowers the "winning" share of the original manager, \( s_1^1 \), so that the worst-off type of the latter, \( a_1^* = G^{-1}\left( \frac{r_1}{s_1^1} \right) \), has a higher ability. Since a manager with high ability expects lower gains from trade, the net surplus from restructuring from his perspective is reduced. In contrast, an increase in the manager’s losing share, \( s_1^0 \), does not affect the identities of the worst-off types of other shareholders and, therefore, does not affect their expected surplus from restructuring. Accordingly, the optimal share rule specifies \( s_1^0(r_1) \geq s_1^0(r_1) \). Similarily, because a larger \( s_1^1 \) yields a smaller \( a_1^* \) (so that this type expects greater gains from restructuring), while larger values of \( \{s_i^1\}_{i\neq 1} \) do not change \( \{a_i^*\}_{i\neq 1} \), the optimal share rule specifies \( s_1^1(r_1) \geq s_1^1(r_1) \).

Note that there is, essentially, an "optimal" worst-off type of manager \( a_w \), given by condition (15). \( V(r_1,s) \) is concave in \( s_1^1 \), so when it is possible to choose \( s_1^1(r_1) \in [\underline{s},1] \) such that (15) holds, then \( a_1^* = a_w \) and we say that \( s_1^1(r_1) \) is interior. When \( \underline{s} = 0 \) and \( r_1 = 0 \), it is impossible to manipulate \( s_1^1 \) to change \( a_1^* \), so the corner solution \( s_1^1(r_1) = \underline{s} \) arises. On the other hand, when \( r_1 \) is large, the value of \( s_1^1 \) that sets \( a_1^* = a_w \) may be infeasibly high, in which case \( s_1^1(r_1) = 1 \) is optimal (and \( a_1^* > a_w \)).

Proposition 2 contributes also to the literature on optimal severance compensation. In our model, side payments are permitted, so one would expect the manager’s severance to include cash to facilitate his participation. However, the optimal share rule allocates also positive share ownership to deposed managers for any \( \underline{s} < 1 \). Thus, our results predict that optimal severance packages specifically include stock compensation. The reason is that ownership shares are particularly effective in making participation of the manager cheaper to achieve. While there is a large literature on golden parachutes,28 we are not aware of any theory with a similar explanation of why stock compensation should necessarily be part of a severance package.

3.3 The impact of ex ante ownership structure

While the distribution of ownership among non-controlling shareholders does not affect the size of \( V(r_1,s(r_1)) \), the ex ante level of managerial ownership, \( r_1 \), determines the severity of the partici-

\(^{25} s_1^1(r_1) \geq s_1^0(r_1) \) implies that \( 1 - \underline{s} - \frac{n-2}{n+1}[1 - s_1^1(r_1)] \geq \frac{1}{s_1^1}[1 - s_1^1(r_1)] \), which holds because \( s_1^1(r_1) \geq \underline{s} \).

\(^{26}\) Since any non-controlling shareholder receives an ex post managerial share \( \underline{s} \), set just large enough to preclude agency costs, we have that \( s_1^1(r_1) \geq s_1^0(r_1) = \underline{s} \).

\(^{27}\) The case \( r_1 = 0, \underline{s} = 0 \), is special because the worst-off type \( a_1^* \) can be any \( a \in [\underline{s}, \bar{s}] \) for the optimal \( s_1^1(0) = 0 \).

\(^{28}\) For example, Almazan and Suarez (2003) show that cash payments for deposed managers can be used in incentive contracts to discipline their behavior. In an empirical examination, Lefanovicz, Robinson and Smith (2000) find that the use of golden parachutes expanded greatly between 1980 and 1995.
pation constraints of non-controlling shareholders and the worst-off type of manager, both directly and through $s(r_1)$ in an optimal mechanism. Hence, $r_1$ is a key determinant of the possibility of efficient restructuring.

Holding $s$ fixed, as in Proposition 1, it is clear from (14) that there are two channels through which managerial ownership directly affects efficiency. First, an increase in managerial ownership $r_1$ slacks the outside shareholders’ participation constraints, increasing $V(r_1, s)$ by a factor of $\mu$ at the margin. Larger initial shares for non-controlling shareholders (and thus lower initial managerial ownership) make it more expensive to induce the participation of low-ability non-controlling shareholders. We refer to this force as the non-controlling shareholder participation effect: it becomes easier to induce the participation of non-controlling shareholders in a mechanism that reallocates control as the initial stake in the hands of insiders increases.

Second, $r_1$ has a negative effect on $V(r_1, s)$ because a higher $r_1$ implies a higher worst-off type $a^*_w$ for the original manager, which in turn reduces the expected gains from restructuring available to bribe that type under the mechanism. Intuitively, the worst-off type of the incumbent manager knows that he will get the least informational rent. Thus, his main incentive to participate is his expectation of sharing some of the efficiency gains through his ex post ownership of shares $s'_{1}$ or $s'_{0}$. But if his ability is high, these expected efficiency gains are small. Thus, as managerial ownership increases, so does management resistance to changes. In line with previous literature, we call this the management entrenchment effect.

Now consider the impact of the optimal share rule. As $r_1$ increases, $s'_{1}(r_1)$ also increases, if possible keeping $a^*_w = a_w$. This reduces the negative effect of $r_1$ on $V(r_1, s(r_1))$. Still, the management entrenchment effect unambiguously dominates.

**Proposition 3** $V(r_1, s(r_1))$ is strictly decreasing in $r_1$.

Thus, efficient restructuring is possible for larger $D$ when $r_1$ is smaller. A lower $r_1$ means that a lower $s'_{1}$ can be used to make $a^*_w = a_w$, leaving more shares to allocate to non-controlling shareholders, thereby decreasing informational rents. In effect, a greater ex ante separation of ownership from control means that a greater ex post separation yields the optimal worst-off type. Intuitively, the direct effect of increasing $r_1$ is equal to the forgone gains from trade for the optimal worst-off type of manager, $a_w$ (differentiate equation 14 with respect to $r_1$, at the optimal share rule with an interior $s'_{1}(r_1)$). Transferring ex ante shares from the non-controlling shareholders to the manager increase the frictions in the mechanism. Increasing $s'_{1}(r_1)$ partially offsets this direct

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29 Proposition 3 does not necessarily hold if $r_1 < a$ and agency costs are modeled directly. See Ferreira et al. (2005) for sufficient conditions for monotonicity of $V(r_1, s(r_1))$ in this case.
effect when it is possible to do so. For larger $r_1$, when $s^1_1(r_1) = 1$, even this partial mitigation of the management entrenchment effect is not possible, so the net surplus falls even faster as $r_1$ rises. Hence, this result suggests that the optimal ex ante managerial ownership sets $r_1$ at the lowest level such that agency costs are precluded, $\underline{s}$.

Let us then define the set of ex ante ownership structures for which efficient restructuring is possible and characterize this set.

**Definition 3** Let $\Phi = \{r_1 | V(r_1, s(r_1)) \geq D\}$ be the set of all $r_1$ for which efficient restructuring is possible.

**Proposition 4** If $\underline{s} = 0$ and $D = 0$, then $\Phi = [0, 1]$.

If there are no agency costs and no direct restructuring costs, efficient restructuring is always possible, because the trivial mechanism will always work. This result yields a subtle, yet important contribution to the broader mechanism design literature. Note that bilateral exchange is a special type of (exogenous) restructuring that emerges in our model when $n = 2$ and both ex ante and ex post managerial ownership are extreme ($r_1 = s^1_1 = s^0_0 = 1$). Myerson and Satterthwaite (1983) show that, under budget balance and symmetric continuous types of buyer and seller, efficient restructuring (or using their language, efficient exchange) is impossible. Their impossibility result hinges, however, on the assumption that ownership and control cannot be traded separably and simultaneously. When we allow the unbundling of ownership and control, the constraint imposed by extreme ownership weakens significantly. The following corollary makes this point by recasting the Myerson-Satterthwaite impossibility result in our setting.

**Corollary 2** Let $r_1 = 1$. In this case, the corporation can be efficiently restructured if and only if $\underline{s} = 0$, $D = 0$, and a share rule that specifies $s^1_1 = s^0_0 = 1$ and $s^1_i = s^0_i = 0$ for $i \neq 1$.

**Proof.** We know from Proposition 2 that the optimal mechanism assigns $s^1_1(1) = 1$. Thus, $V(r_1, s(1))$ collapses to

$$V(1, s(1)) = -\underline{s}(n - 1) \int_{\underline{a}}^{\bar{a}} [1 - F(u)] G(u)du.$$  

Clearly, $V(1, s(1)) \leq 0$, and is strictly negative unless $\underline{s} = 0$. Therefore, efficient restructuring is possible if and only if $\underline{s} = 0$ and $D = 0$.  

Hence, it is possible to efficiently restructure even when $r_1 = 1$, if one employs the trivial mechanism and provided that the complete separation of control from ownership introduces no
agency costs and entails no direct costs of restructuring. Note that, for $s = 1$ and $n = 2$, $V(1, s(1))$ collapses to the minimum outside subsidy required to implement efficient bilateral exchange in the Myerson-Satterthwaite setting. Indeed, their setting can be interpreted as a nested, special case of our model when $r_1 = 1$ and $s = 1$. Since $V(r_1, s)$ is increasing in $s_0$, it then follows that, when $s < 1$ and control and ownership are separably tradeable, efficient bilateral exchange requires a smaller outside subsidy than the one Myerson and Satterthwaite identify, as long as the “seller” (manager) retains some ownership shares when the “buyer” (a non-controlling shareholder) assumes control ex post.

This set of results gives a novel perspective on the conflict between the moral hazard problem and the adverse selection problem. A low level of ex ante managerial ownership is bad for incentives but good for the operation of the market for corporate control, so it mitigates the adverse selection problem but generates a moral hazard problem. For sufficiently high $r_1$, when the market for control fails to yield efficiency, second-best alternatives can dictate keeping a less-qualified manager who will not divert profits or recruiting a more qualified manager who will extract some private benefits. While each problem in isolation can be overcome through the market for corporate control, we show that, jointly, private benefits and asymmetric information create endogenous frictions that sometimes cannot be overcome. Most importantly, they make managerial ownership the crucial determinant of the efficiency of this market and, hence, a key determinant of firm value.

4 Conclusion

The optimal ownership structure targeted by a restructuring mechanism must represent a compromise between two conflicting goals: providing incentives to new managers to maximize firm value, and reducing incumbent management resistance to change. Unless agency costs are extreme, the separation of ownership from control enhances efficiency in the market for control. The reason is that informational rents are smaller when shareholders do not have to surrender all of their shares in submitting to a restructuring mechanism.

This result obtains in a model without financing constraints or risk aversion. As a result, the force encouraging the separation of ownership from control here does not depend on limited entrepreneurial wealth or on diversification economies. Rather, it stems from private information. Thus, instead of replacing existing theories finding that the separation is an efficient, endogenous response to economic primitives, the model developed here augments and reinforces those theories.

We keep the model simple, and this helps to highlight the fundamental, qualitative nature of

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the economics driving the results. For example, contracting on profits is restricted to linear share rules, yet the separation of ownership from control nonetheless emerges. With adjustments to the production technology, it would be possible to analyze additional, non-linear possibilities for share rules, such as stock options. With such additional alternatives, future research may identify more sophisticated ways of reducing frictions in the market for corporate control by further separating ownership from control.

Such innovations to our model may also improve its ability to explain the details of existing mechanisms of transferring control. At this point, it would be misleading to use our results to predict the outcomes of hostile mechanisms, such as proxy fights or tender offers. There is a large literature that focuses on modeling and assessing the efficiency properties of specific mechanisms—e.g. Grossman and Hart (1980), Shleifer and Vishny (1986), Burkart (1995), Singh (1998), Bulow et al. (1999), Burkart et al. (2000), and Mueller and Panunzi (2004). By contrast, in telling us what all specific mechanisms cannot achieve, our approach helps to explain the bounds and limits of the mechanisms that form the market for corporate control. Indeed, one will never be able to fully eliminate the joint problems of management entrenchment and the lack of managerial incentives to maximize profits as long as information asymmetries remain in place.

The techniques of mechanism design offer clear methodological value to the practice of theoretical corporate finance. Moreover, private information and adverse selection are fundamental in financial transactions, as evidenced by the billions of dollars spent every year on "due diligence." Thus, we believe that our approach of unbundling ownership and control and treating them as separate assets has the potential to pave the way for applications in various other related settings as well.

A Appendix

Proof of Lemma 1. Since the assigned shares always have to satisfy budget balance, we have

\[ s_i^1 = 1 - \sum_{j \neq i} s_j^0. \]  

This implies that

\[ \sum_{i \neq 1} s_i^1 = (1 - s_1^0 - s_2^0 - s_3^0 - ...) + (1 - s_1^0 - s_2^0 - s_3^0 - ...) + (1 - s_1^0 - s_2^0 - s_3^0 - ...) + \ldots \]

\[ = (n - 1) (1 - s_1^0) - (n - 2) \sum_{j \neq 1} s_j^0 = (n - 1) (1 - s_1^0) - (n - 2) (1 - s_1^0). \]
Since $s_i^1 \geq a$ for all $i$, it follows that

$$\sum_{i \neq 1} s_i^1 = (n - 1) (1 - s_i^0) - (n - 2) (1 - s_i^1) \geq (n - 1)a.$$  

From condition (16), we have also that

$$s_i^1 - s_i^0 = 1 - \sum_{j \neq i} s_j^0 - s_i^0 = 1 - \sum_{j} s_j^0.$$  

Aggregating over all $i \neq 1$ and using condition (16) once more, we then obtain

$$\sum_{i \neq 1} (s_i^1 - s_i^0) = (n - 1)(1 - \sum_{j} s_j^0) = (n - 1) (s_i^1 - s_i^0),$$  

(18) completing the proof.

**Proof of Lemma 2.** From (8) we have that net utility $U_1(a_1, a_1)$ is convex and has first derivative $s_1^1G(a_1) - r_1$. Thus, if $s_1^1 \geq r_1$ and $s_1^1 > 0$, $a_1^*$ is identified by the first-order condition

$$a_1^* = G^{-1}\left(\frac{r_1}{s_1^1}\right).$$  

If $s_1^1 < r_1$, then $s_1^1G(a_1) - r_1 < 0$ for all $a_1 \in [a, a]$, and $U_1(a_1, a_1)$ is minimized at $a_1^* = \bar{a}$. Finally, if $s_1^1 = r_1 = 0$, all $a_1 \in [a, a]$ expect the same gain under the mechanism, so any type can be considered the worst-off type.

Participation is individually rational for all types of shareholder 1 if and only if $U_1(a_1^*, a_1^*) \equiv s_1^1\left[a_1^*G(a_1^*) + \int_{a_1^*}^{\bar{a}} uG(u)\right] - k_1 - r_1 a_1^* \geq 0$. Thus, if $s_1^1 \geq r_1$ and $s_1^1 > 0$, the restriction simplifies to

$$k_1 \leq s_1^1 \int_{a_1^*}^{\bar{a}} uG(u).$$  

This yields condition (11), since $\max\{ (r_1 - s_i^1) \bar{a}, 0 \} = 0$ when $s_i^1 \geq r_1$. If $s_i^1 < r_1$, $a_1^* = \bar{a}$ and participation is individually rational for the original manager if and only if

$$k_1 \leq -(r_1 - s_i^1)\bar{a}.$$  

This also yields condition (11), since $s_i^0 \int_{\bar{a}}^{\bar{a}} uG(u) = 0$. When $s_i^1 = r_1 = 0$, condition (11) is trivially satisfied.

For all subsequent proofs, let $\{a_i^*\}$ be defined as in lemmas 2 and 3.

**Proof of Lemma 3.** For $i \in \{2, ..., n\}$, we know from (8) that net utility $U_i(a_i, a_i)$ is strictly convex with first derivative $s_i^1G(a_i)$. Thus, for $s_i^1 > 0$, it is increasing in $a_i$ for all $a_i \geq a$. Hence
$a_i^* = a$ and participation is individually rational for all types of shareholder $i \in \{2, ..., n\}$ if and only if $U_i(a, a) = s_i^1 \int_a^\bar{u} udG(u) - k_i - r_i u \geq 0$, which is equivalent to condition (12) because $a_i^* = a$.

If $s_i^1 = 0$, the first derivative of $U_i(a_i)$ is zero. Hence all $a_i \in [a, \bar{a}]$ expect to obtain the same net utility from the mechanism, so any type can be considered the worst-off type. In this case, $U_i(a_i^*, a_i^*) = -k_1 - r_i u \geq 0$, which is equivalent to (12) because $s_i^1 = 0$. ■

**Proof of Proposition 1.** Only if. In a mechanism that satisfies (2), the transfers defined in (6) imply that

$$T_i (a_i) - T_i (b) = (s_i^1 - s_i^0) \int_{a_i}^{\bar{a}} udG(u) - (s_i^1 - s_i^0) \int_b^{a_i} udG(u)$$

so we have that $T_i (a_i) = T_i (a_i^*) - (s_i^1 - s_i^0) \int_{a_i}^{\bar{a}} udG (u)$. Taking ex ante expectations, we obtain

$$E_i \{T_i(a_i)\} = E_i \{T_i(a_i^*)\} - (s_i^1 - s_i^0) \int_{a_i}^{\bar{a}} \int_{u=a_i}^{u=a_i^*} udG(u)dF(a_i)$$

$$= T_i(a_i^*) - (s_i^1 - s_i^0) \left( \int_{u=a_i^*}^{\bar{a}} dF (a_i) udG(u) - \int_{u=a}^{u=a_i^*} dF (a_i) udG(u) \right)$$

$$= T_i(a_i^*) - (s_i^1 - s_i^0) \left( \int_{a_i^*}^{u} [1 - F (u)] udG(u) - \int_{a_i^*}^{a} F (u) udG(u) \right)$$

$$= T_i(a_i^*) - (s_i^1 - s_i^0) \left( \int_{a_i^*}^{u} udG(u) - \int_{a}^{a_i^*} F (u) udG(u) \right),$$

where the second line is obtained by changing the order of integration. Because of budget balance, we have that $\sum_{i=1}^{n} E_i \{T_i(a_i)\} = -D$. Thus,

$$\sum_{i=1}^{n} T_i(a_i^*) = \sum_{i=1}^{n} \left[ (s_i^1 - s_i^0) \left( \int_{a_i^*}^{u} udG(u) - \int_{a}^{a_i^*} F (u) udG(u) \right) \right] - D. \tag{20}$$

But from expression (6) defining the transfers, we also know that

$$\sum_{i=1}^{n} T_i(a_i^*) = \sum_{i=1}^{n} (s_i^1 - s_i^0) \int_{a_i^*}^{u} udG(u) - \sum_{i=1}^{n} k_i. \tag{21}$$

Substituting (20) in (21), we then have that

$$\sum_{i=1}^{n} \left[ (s_i^1 - s_i^0) \left( \int_{a_i^*}^{u} udG(u) - \int_{a}^{a_i^*} F (u) udG(u) \right) \right] - D = \sum_{i=1}^{n} (s_i^1 - s_i^0) \int_{a_i^*}^{u} udG(u) - \sum_{i=1}^{n} k_i,$$
which can be re-organized as follows:

$$\sum_{i=1}^{n} k_i = \sum_{i=1}^{n} \left( (s_i^1 - s_i^0) \int_{\underline{a}}^{\bar{a}} F(u)udG(u) \right) + D. \tag{22}$$

Now, aggregating individual rationality constraints (11) and (12), they require that

$$\sum_{i=1}^{n} k_i \leq \sum_{i=1}^{n} s_i^1 \int_{\underline{a}}^{\bar{a}} udG(u) - \max\{(r_1 - s_i^1)\bar{a}, 0\} - (1 - r_1)\mu, \tag{23}$$

since $$\sum_{i\neq 1} r_i = 1 - r_1$$. Using (22), it follows immediately that this inequality is equivalent to (14).

If. Condition (2) guarantees that the mechanism assigns control to the shareholder with the highest announced ability. We have showed in the text that the transfers defined in (6) ensure incentive compatibility. Thus, it remains to be shown only that the mechanism is individually rational and ex ante budget balanced.

Aggregating the individual rationality constraints require (23) to be satisfied. But (13) implies that (23) is satisfied when the $$k_i$$ are defined so that (22) holds, which the only if part of the proof shows to be compatible with the definition of transfers in (6).

**Proof of Proposition 2.** We first identify $$s(r_1)$$, then prove the “if and only if” statement. In identifying $$s(r_1)$$, it is easiest to first prove part iii, followed by parts ii, i and iv.

(Part iii) Consider first the manager’s golden parachute, $$s_i^0$$. The budget balance conditions of Lemma 1 yield substitutions of $$\sum_{i\neq 1} s_i^1$$ and $$\sum_{i\neq 1} (s_i^1 - s_i^0)$$ out of expression (14), so $$V(r_1, s)$$ may be rewritten as a function of $$s_i^1$$ and $$s_i^0$$ only:

\[
V(r_1, s) = s_i^1 \int_{\underline{a}}^{\bar{a}} udG(u) + [(n - 1)(1 - s_i^0) - (n - 2)(1 - s_i^1)] \int_{\underline{a}}^{\bar{a}} udG(u) - n (s_i^1 - s_i^0) \int_{\underline{a}}^{\bar{a}} F(u)udG(u) - (1 - r_1)\mu - \max\{(r_1 - s_i^1)\bar{a}, 0\}. \tag{24}
\]

We then have

\[
\frac{dV(r_1, s)}{ds_i^0} = \int_{\underline{a}}^{\bar{a}} F(u)udG(u) - (n - 1) \int_{\underline{a}}^{\bar{a}} [1 - F(u)]udG(u)
\]

\[
= (n - 1) \int_{\underline{a}}^{\bar{a}} [1 - F(u)] G(u)du > 0, \tag{25}
\]

where the second line uses the fact that $$G(u) = F(u)^{n-1}$$. Clearly, then, net surplus is maximized for the largest possible $$s_i^0$$ that is compatible with $$s_i^1(r_1)$$ and with the budget balance condition from Lemma 1:

\[
\sum_{i\neq 1} s_i^1 = (n - 1)(1 - s_i^0) - (n - 2)(1 - s_i^1) \geq (n - 1)\underline{a}. \tag{26}
\]
The left-hand side of this inequality is positive when \( s^0_i = 0 \) and decreases in \( s^0_i \). Thus, \( V(r_1, s) \) is maximized when the constraint holds with equality, implying that the optimal golden parachute is

\[
s^0_1(r_1) = 1 - \frac{2}{n} \left[ 1 - s^1_1(r_1) \right],
\]

where \( s^1_1(r_1) \) is also chosen optimally.

(Part \( ii \)) To find \( \{s^i_1(r_1)\}_{i \neq 1} \), note that part \( iii \) also implies that \( \sum_{i \neq 1} s^i_1 = (n - 1)\underline{s} \). Since each \( s^i_1 \) must be at least \( \underline{s} \), we have that \( s^i_1(r_1) = \underline{s} \) for \( i \neq 1 \).

(Part \( i \)) To find \( \{s^0_i(r_1)\}_{i \neq 1} \), rewrite (16) isolating the summation on the left-hand side for each \( i \neq 1 \) that may gain control. This gives us the following \( n - 1 \) conditions:

\[
\begin{cases}
  s^0_1(r_1) + s^0_3 + s^0_4 + \ldots + s^0_n = 1 - \underline{s} \\
  s^0_1(r_1) + s^0_2 + s^0_4 + \ldots + s^0_n = 1 - \underline{s} \\
  \vdots \\
  s^0_1(r_1) + s^0_2 + s^0_3 + \ldots + s^0_{n-1} = 1 - \underline{s} ,
\end{cases}
\]

where we have used the fact that \( s^i_1(r_1) = \underline{s} \) for all \( i \neq 1 \) in the optimal mechanism. It is easy to see that these conditions are only satisfied for

\[
s^0_2 = s^0_3 = \ldots = s^0_n .
\]

Using this condition, and substituting (27) into one of the \( n - 1 \) conditions above, we find that

\[
s^0_i(r_1) = \frac{1 - s^1_1(r_1)}{n - 1}
\]

for all \( i \neq 1 \).

(Part \( iv \)) Next consider the optimal \( s^1_1 \). Using the results from parts \( i \) to \( iii \) of the proposition, we can substitute into (24) to yield the function \( V'(r_1, s^1_1) \):

\[
V'(r_1, s^1_1) = s^1_1 \int_{a^*_1}^{a} udG(u) + \left[ 1 - s^1_1 - (n - 1)\underline{s} \right] \int_{a}^{a^*} udF(u)^n + (n - 1)\underline{s} \int_{a}^{a^*} udG(u) \\
- (1 - r_1)\mu - \max\{ (r_1 - s^1_1)a, 0 \} .
\]

The \( s^1_1 \) that maximizes this expression defines the optimal \( s^1_1(r_1) \).

Assume, for the moment, that \( \underline{s} = 0 \). If \( r_1 = 0 \) and \( s^1_1 > 0 \), then \( a^*_1 = \underline{s} \) and

\[
\frac{\partial V'(r_1, s^1_1)}{\partial s^1_1} = \int_{a}^{a^*} udG(u) - \int_{a}^{a^*} udF(u)^n < 0,
\]

so \( s^1_1(r_1) = 0 \) if \( r_1 = 0 \) and \( \underline{s} = 0 \) (this follows from the continuity of \( V' \)).

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For positive \( r_1 \) if \( z = 0 \), or for all \( r_1 \) if \( z > 0 \), we will show that it is optimal to have \( s_1^r > r_1 \geq r_1 \) in this case. Note first that, since \( s_1^r \) is defined on \([z, 1]\), a compact set, if \( V'(r_1, s_1^r) \) is bounded then it must achieve a maximum on this set. To show that the optimal \( s_1^r \) is unique and greater than \( r_1 \), it suffices to show that \( V'(r_1, s_1^r) \) is strictly increasing in \( s_1^r \) when \( s_1^r \leq r_1 \), strictly concave for \( s_1^r > r_1 \), and has a continuous first derivative and is finite when \( s_1^r = 1 \).

First, when \( s_1^r \leq r_1 \), we have

\[
\frac{\partial V'(r_1, s_1^r)}{\partial s_1^r} = \bar{a} - \int_a^\bar{a} udF(u) > 0,
\]

so \( V'(r_1, s_1^r) \) is increasing and linear in \( s_1^r \) in this range. Next, when \( s_1^r > r_1 \), we have

\[
\frac{\partial V'(r_1, s_1^r)}{\partial s_1^r} = a_1^* G(a_1^*) + \int_{a_1^*}^{\bar{a}} udG(u) - \int_{a_1^*}^{\bar{a}} udF(u) = \bar{a} - \int_{a_1^*}^{\bar{a}} G(u)du - \int_{a}^{\bar{a}} udF(u) > 0,
\]

where the second line follows from a simple integration by parts. Since \( a_1^* = \bar{a} \) at \( s_1^r = r_1 > 0 \), this expression collapses to (29) at that point, so the derivative is continuous at \( s_1^r = r_1 \). It is straightforward to show that \( V'(r_1, s_1^r) \) is strictly concave for \( s_1^r > r_1 \):

\[
\frac{\partial^2 V'(r_1, s_1^r)}{\partial (s_1^r)^2} = G(a_1^*) \frac{da_1^*}{ds_1^r} < 0.
\]

Finally, since \( V'(r_1, s_1^r) \) is finite when \( s_1^r = 1 \), if there exists an \( s_1^r \in [z, 1] \) such that \( \frac{\partial V'(r_1, s_1^r)}{\partial s_1^r} = 0 \), then that \( s_1^r \) is the unique maximizer of \( V'(r_1, s_1^r) \). In that case, the optimal \( s_1^r \) is defined implicitly by the first-order condition (15). Otherwise, either

\[
\bar{a} - \int_{G^{-1}(r_1)}^{\bar{a}} G(u)du - \int_{a}^{\bar{a}} udF(u) > 0,
\]

in which case \( s_1^r = 1 \) is the unique maximizer, or

\[
\bar{a} - \int_{G^{-1}(r_1)}^{\bar{a}} G(u)du - \int_{a}^{\bar{a}} udF(u) < 0,
\]

which is impossible if \( r_1 \geq z \). This completes our identification of \( s(r_1) \).

To show that a corporation with initial managerial ownership \( r_1 \) can be efficiently restructured if and only if \( V(r_1, s(r_1)) \geq D \), note that if \( V(r_1, s(r_1)) \geq D \), the corporation can be efficiently restructured using \( s(r_1) \), because \( s(r_1) \) satisfies the budget balance conditions in lemmas 3 and 1, so there are no agency costs ex post, and \( V(r_1, s(r_1)) \geq D \) implies that \( s(r_1) \) yields an efficient
transfer of control by Proposition 1. On the other hand, if the corporation can be efficiently restructured, then \( V(r_1, s) \geq D \) for some \( s \) that satisfies the budget balance conditions in (??). Since \( V(r_1, s(r_1)) \geq V(r_1, s) \) by the definition of \( s(r_1) \), we have that \( V(r_1, s(r_1)) \geq D \). Since \( s(r_1) \) satisfies budget balance as well, the corporation can be efficiently restructured using share rule \( s(r_1) \).

**Proof of Proposition 3.** To prove our results, we begin by showing that \( a_w > \mu \).

Consider the term \( \int_{a}^{\hat{a}} udF(u) \). Consider a random variable \( X \) that follows \( F \) and a random variable \( Y \) that follows \( G \). Define the random variable \( Z \) as

\[
Z = \max\{X, Y\}.
\]

Thus,

\[
E[Z] = \int_{a}^{\hat{a}} ud[G(u)F(u)].
\]

The expected \( Z \) conditional on already knowing \( a \) is

\[
h(a) = E[Z | a] = aG(a) + \int_{a}^{\hat{a}} udG(u).
\]

By the law of iterated expectations, we then have

\[
E[Z] = E_{a}[E[Z | a]] = E_{a}[h(a)].
\]

Thus, we can conclude that

\[
E_{a}[h(a)] = \int_{a}^{\hat{a}} ud[G(u)F(u)].
\]

Now let us go back to the first-order condition for an optimal \( s^1 \). Suppose we are in an interior optimum. The problem is to find \( a_w \) such that

\[
a_wG(a_w) + \int_{a_w}^{\hat{a}} udG(u) - \int_{a}^{\hat{a}} ud(G(u)F(u)) = 0,
\]

or equivalently,

\[
h(a_w) - E_{a}[h(a)] = 0.
\]

Notice that \( h(a) \) is increasing and convex. Thus, Jensen’s inequality implies that

\[
h(\mu) - E_{a}[h(a)] < 0,
\]

where \( \mu = E[a] \), thus the value \( a_w \) that solves

\[
h(a_w) - E_{a}[h(a)] = 0
\]
is such that \( a_w > \mu \).

Now, differentiating \( V(r_1, s(r_1)) \) with respect to \( r_1 \), we obtain

\[
\frac{dV(r_1, s(r_1))}{dr_1} = \frac{\partial V}{\partial r_1} + \frac{\partial V}{\partial s_1} \frac{\partial s_1}{\partial r_1} = \mu - a^*_1
\]

where the envelope theorem zeroes out the last term in the first line of algebra above. Since \( r_1 \geq s \), then it is clearly true that

\[
\frac{dV(r_1, s(r_1))}{dr_1} < 0
\]

**Proof of Proposition 4.** Suppose \( s = 0 \) and \( D = 0 \). It suffices to show that the trivial mechanism, where \( s_1^i = s_0^i = r_i \) for all \( i \), achieves ex post efficiency for all \( r_1 \). Since \( s = 0 \), agency costs never arise. Since under the trivial mechanism \( s_1^i = s_0^i \) for all \( i \), we have, using equation (20) from the proof of Proposition 1, that

\[
\sum_{i=1}^{n} T_i(a^*_i) = \sum_{i=1}^{n} \left( s_1^i - s_0^i \right) \left( \int_{a_i^*}^{a} udG(u) - \int_{a}^{a} F(u)udG(u) \right) = 0.
\]

Individual rationality requires that equation (23) hold:

\[
\sum_{i=1}^{n} T_i(a^*_i) = 0 \geq \left[ \max\{(r_1 - s_1^i)\tilde{a}, 0\} + (1 - r_1)\mu - \sum_{i=1}^{n} s_0^i \int_{a_i^*}^{a} udG(u) \right].
\]

Given the trivial mechanism’s share rule, \( a^*_i = \pi \) and the right-hand side of the above expression becomes

\[
(1 - r_1) \left[ \mu - \int_{a}^{a} udG(u) \right] \leq 0,
\]

so that \( V(r_1, s^i) \geq 0 \) for any \( r_1 \), where \( s^i \) is the share rule for the trivial mechanism. Since \( V(r_1, s(r_1)) \geq V(r_1, s^i) \), the proof is complete.

References


