1. The on-going conflict in Iraq has, without question, increased overall spending of the US government, as well as the federal budget deficit. A recent NY Times article (4/30/04) claims that this spending has significantly contributed to the rise in US real GDP over the first quarter of 2004. (Don’t use math in your answer.)
   a. In the context of the basic macro framework discussed in class, under what conditions will the article’s claim be correct; i.e. that this (exogenous) increase in spending, holding tax revenues fixed, will cause an increase in aggregate output? Give a specific example of the mechanism by which this effect could occur.
   b. In the basic model, under what conditions will the increase in government spending have no effect on real GDP? Briefly explain.
   c. Suppose that President Kerry, upon taking office, raises tax revenues to finance expenditures on the war. Under what conditions will this tax policy, as opposed to borrowing, be ‘irrelevant’ to aggregate output, consumption and investment?

2. A value-maximizing firm facing convex adjustment costs to investment will satisfy the following Euler equations for each period $t$ in the planning period:
   \[ k_{t+1} - k_t = 0.5(q_t - 1) \]
   \[ (1 + r)q_t = f'(k_{t+1}) + q_{t+1} \]
   where $k$ is the capital stock, $q$ is the shadow marginal value of capital, $f'(\cdot)$ is the marginal product of capital (which exhibits diminishing returns), and the rate of depreciation is 0.
   a. Draw the phase diagram illustrating the dynamics of $q$ and $k$ implicit in this model. Be sure to include the steady-state, the saddlepath, and a complete description of how $q$ and $k$ evolve over time when beginning away from the steady-state.
   b. Suppose the real interest rate increases. Describe the effect on the steady-state, and on the transition to the steady-state.

3. Real Business Cycle theories explain the dynamics of aggregate fluctuations as the optimal response of output to real shocks. All other theories of the cycle assume some form of ‘imperfection’ or ‘market-rigidity.’ For each of the following models, explain (in one paragraph, without equations) i) the nature of the imperfection or rigidity; ii) what feature of the cycle the model helps explain; and iii) how it explains this feature.
   a. Lucas’ island model.
   b. An efficiency wage model.
   c. A sticky-price model with ‘menu costs’.
4. Consider the following model of aggregate output and the price level (in logs):

\[ y_t = a_0 + a_1(p_t - p_t^e), \quad \text{(aggregate supply)} \]
\[ y_t = b_0 + b_1(M_t - p_t), \quad \text{(aggregate demand)} \]
\[ p_t^e = p_{t-1} \]

where \( y \) is output, \( p \) is the price level, \( M \) is nominal money, and \( p^e \) is suppliers’ expected price level. All parameters are positive.

a. Can we characterize expectations in this model as ‘rational?’ Why or why not?

b. Show that the equilibrium price level can be expressed as a first-order difference equation.

c. Provide a general expression for the dynamic multiplier \( \frac{\partial y_{t+1}}{\partial M_t} \).

d. What evidence did Sargent claim did not support this model of expectation-formation?

5. Economic growth.

a. Suppose a politician calls for policies to increase the saving rate of his economy to promote sustained economic growth. Carefully explain why Solow’s model of growth implies that this policy will not achieve the objective of sustained growth rates for output.

b. No matter how much the media claim otherwise, Alan Greenspan does not set interest rates, at least not in the long-run, and not in real terms. Use Ramsey’s model of growth to describe how the real interest rate is determined in the long-run.
Answers

1. War spending.
   a. In the basic model exogenous government spending will lead to an increase in current output only if there is some market imperfection or rigidity. For example, nominal wages or prices could be sticky, so that labor or goods markets are slow to clear, or agents may have imperfect information. Suppose that nominal wages are sticky, for institutional reasons (e.g. long-term contracts). An increase in government spending, *ceteris paribus*, will increase the demand for goods, thereby raising interest rates. Higher interest rates will reduce the demand for real money balances; the price level will rise so that real money falls. The rising price level will cause real wages to fall, since the nominal wage is fixed, which provides an incentive for firms to increase output due to a decrease in the marginal cost of labor.

b. If prices in all markets are flexible so that they always clear, government spending will have no effect on aggregate output. Here’s why: as before, the increased government spending increases the interest rate and the price level. However, nominal wages will respond proportionately to the price level increase, so that real wages remain unchanged. Firms therefore have no incentive to accommodate the increase in demand, and output will remain unchanged. The continued excess aggregate demand will lead to an even greater increase in the interest rate (compared to the sticky wage model), so that consumption and investment will fall (in sum) by the same amount that government spending is rising. Thus, gov’t spending will totally crowd out private spending, so that neither output nor aggregate spending change in equilibrium.

c. If people in the economy foresee that Kerry’s current tax increase is simply a reduction in the present value of future taxes, they will reduce saving by the amount of the tax increase (since future taxes will not be as high as expected, there is less of a need to save), and consumption will be unaffected. Even though the government budget deficit falls, national saving will not change, so there is no pressure on interest rates or demand. Under these conditions of ‘Ricardian Equivalence’, the equilibrium outcome for output, consumption and investment will be the same whether Kerry increases taxes or increases debt to pay for the war (assuming the level of spending does not change) – bond finance is equivalent to tax finance.

2. Optimal investment
   a. Let \( q \) be measured along the vertical axis, and \( k \) is on the horizontal. The first Euler equation implies that capital is constant when \( q = 1 \), for any \( k \). The second Euler equation implies that \( q \) is constant, given \( k \), when \( q = f'(k) \). Since \( f''(k) < 0 \), this locus will be downward sloping. The intersection of these loci defines the steady-state values of \( q(= 1) \), and \( k = (r/\alpha)^{1/(\alpha-1)} \) for \( f(k) = k^{\alpha} \). The first equation implies that if \( q > 1 \), net investment will be positive, so that capital will increase. The second equation implies that, for a given value of \( q \), if \( k \)
is less than the value defined by the downward sloping locus, $q$ must fall (given $q_t = q_{t+1}$ initially, if $k$ falls, $f'(k)$ rises, and $q_{t+1}$ must fall below $q_t$). These equations thus determine the laws of motion of $q$ and $k$ outside the steady-state. The model implies a downward sloping saddlepath, which for a given value of $k$ defines a unique value of $q$ such that these variables will approach the steady-state over time. Note that only the saddlepath is consistent with the transversality condition.

b. A rise in the interest rate will shift the $(q_t = q_{t+1})$ locus to the left (actually, in this case, it stretch the locus towards the origin). At the same time, the saddlepath will shift downwards. At the instant of the increase in $r$, $q$ falls immediately to the new saddlepath, and the stock of capital remains at its original level. Since $q$ falls below 1, and since capital is to the right of the downward sloping locus, capital will gradually fall and $q$ will rise until the new steady-state is obtained. In this steady-state, capital will be lower than its initial value, and $q = 1$.

   a. Lucas’s island model: the rigidity is imperfect information about aggregate shocks; in particular, the price level and the money supply. The model can explain the observation that money and output are positively correlated over the cycle. The model assumes that producers respond positively to their perceived relative price. Suppose the aggregate money supply increases, but that this increase is unknown to producers on the scattered islands. Each producer observes an increase in demand, and thus price, for his or her respective good. From this price, the producer tries to predict the price level in order to determine relative price. Thinking (mistakenly) that this increase at least partially reflects an increase in relative price, each producer responds (rationally) by increasing output. The overall effect is that the increase in money causes an increase in aggregate output. In addition, the model implies that the output response to an aggregate demand shock becomes smaller as the variance of aggregate demand shocks becomes larger.

   b. Efficiency wage models assume that there is a benefit to firms of raising wages, not just a cost. One reason for this is incomplete, or asymmetric information. The model implies a rigidity to real wages, since they are independent of the supply of labor even in equilibrium. These models can explain ‘involuntary’ unemployment as an equilibrium result. Suppose that a negative aggregate demand shock leads to a decrease in the demand for labor (due to an unspecified nominal rigidity), and an excess supply of labor. Even though workers are willing to work at a lower wage, firms have no incentive to offer lower wages, because this will reduce the quality of labor. Thus, wages remain fixed, and layoffs occur.

   c. Menu costs are the fixed costs associate with changing prices. They are usually considered in models in which firms have some market power, so that they are price setters, not price takers. Models with menu costs can rationalize sticky goods prices, and explain why firms respond to shocks to aggregate demand by adjusting quantity, not price. If the benefits of adjusting price are lower than the menu costs, then fixed prices will be an equilibrium in the face of demand shocks.
Even small menu costs can lead to fix prices: if costs don’t respond much to changes in demand (i.e. there is a real rigidity), the benefits of price adjustment will be small.

4. Dynamic macro model.
   a. Expectations are not rational; they are adaptive, since the expectation is set simply to last period’s price, not the expectation implied by the model.
   b. Equate supply and demand, use the third equation, and solve for \( p_t \):

\[
\begin{align*}
a_0 + a_1(p_t - p_{t-1}) &= b_0 + b_1(M_t - p_t) \\
p_t &= \frac{b_0 - a_0}{a_1 + b_1} + \frac{a_1}{a_1 + b_1}p_{t-1} + \frac{b_1}{a_1 + b_1}M_t \\
&= \alpha_0 + \alpha_1p_{t-1} + (1 - \alpha_1)M_t
\end{align*}
\]

   c. Dynamic multiplier:

\[
\begin{align*}
\frac{\partial y_t}{\partial M_t} &= b_1\alpha_1 = \frac{b_1a_1}{a_1 + b_1} \\
\frac{\partial y_{t+k}}{\partial M_t} &= -b_1(1 - \alpha_1)\alpha_1^i, \quad i = 1, 2, \ldots
\end{align*}
\]

d. Sargent’s examination of the ends of four big inflations in Europe in the 1920’s suggested that inflation ended quickly, with less than expected loss in output. He claims that this pattern is inconsistent with models assuming adaptive expectations, since such expectations impose momentum in inflation and inflationary expectations. In this case, policies to reduce aggregate demand can only gradually reduce inflation, and output will fall in the short-run.

5. Economic growth.
   a. One key feature of Solow’s model of growth is that there are diminishing returns to capital. This implies that in the steady-state, capital per worker will grow at the exogenous rate of growth of technology. Suppose that saving rates are increased through government policies. In the short-run, output will grow at a fast pace, given the increase in the rate of capital accumulation. However, as capital grows, its marginal product falls, reducing incentives to accumulate more capital, and growth will slow. Ultimately, the increased growth rate will converge to its long-run level. Because of diminishing returns to capital, sustained growth depends only on the rate of increase of technology, not the rate of saving.

   b. In both the Solow and Ramsey models, the real rate of interest is defined to be the marginal product of capital (this is because implicity markets are competitive; also, think of investment when \( q \) always equals 1). The Ramsey model of optimal growth implies that the time path of consumption is optimal when the intertemporal marginal rate of substitution between future and current consumption equals the reciprocal of the marginal product of capital. This condition implies that in the the long-run steady-state, when consumption is constant, the
household’s discount factor equals the reciprocal of the interest rate. Thus, the stock of capital adjusts until the real interest rate equals households’ rate of time preference. The real interest rate is ultimately determined by our preferences for intertemporal consumption (assuming no technology growth). (If the capital stock is low, the return to capital accumulation – saving – is high. Thus, when the capital stock is low, saving is high and future consumption is low. But as capital accumulates, its return falls, and future consumption will begin to rise. This will continue until the marginal costs of changing future consumption for current consumption – the household’s discount rate – equals the marginal benefits – the return to capital.