1. Consider an exogenous increase in government spending fully financed by an increase in lump sum taxes (i.e., \( dg = dT \)). Compute and determine the sign of the reduced form multipliers of output and the nominal interest rate for this fiscal policy experiment using the following models developed in class: a) the “sticky-wage” model and b) the “sticky-price” model. For each model, use the following expression for aggregate demand: \( y = c(y - T, r) + i(r) + g \). For each case, briefly explain and interpret these multipliers in terms of the implied economic behavior.

2. Assume the macro economy is described by the following equations,

\[
\begin{align*}
\frac{w}{p} &= f_n(N) \\
N &= N(\frac{w}{p^e}) + z \\
y &= f(N) \\
y = y(r - \pi, g), \quad y_1 < 0, y_2 > 0 \\
\frac{M}{p} &= m(r, y) \\
p^e &= h(p), \quad 0 \leq h'(p) \leq 1.
\end{align*}
\]

The variables and equations are as defined in class. The endogenous variables are \( w, p, N, y, r, \) and \( p^e \). \( z \) is an exogenous variable that increases the supply of labor, \( ceteris paribus \). To make life simpler, assume that in initial equilibrium \( p = h(p) = w = M = 1 \).

a. The aggregate supply curve is defined as the relationship between output \( (y) \) and the price level \( (p) \) such that the labor market is in equilibrium and firms are producing output according to the production function. Derive the aggregate supply curve for this model after linearizing the system by taking total differentials. (Hint: appropriately combine the first three and last equations.) Explain the economic behavior underlying this supply curve.

b. Compute the reduced form effects of a change in \( g \) (government spending) on the equilibrium values of \( y, r, p \) (Hint: if you have done (a) correctly, you should have a three equation system in these variables). Are these multipliers positive, negative, or zero? Explain the underlying economic behavior.

c. Compute the multiplier \( \frac{\partial r}{\partial g} \) for the flexible price (full employment) model (by appropriately restricting \( h'(p) \)). Show that this multiplier is larger in part (c) than in part (b). Explain why.

d. Compute the reduced form multipliers \( \frac{\partial y}{\partial z} \) and \( \frac{\partial r}{\partial z} \) for the general case. Explain.
Answers

1. Balanced budget multipliers. In each of the answers, I assumed that $M = w = p = 1$ as initial equilibrium values for convenience.
   a. Sticky-wage model.

   \[
   \begin{bmatrix}
   1 & 0 & \frac{f_n}{f_{nn}} \\
   (1 - c_1) & -(i' + c_2) & 0 \\
   m_y & m_r & 1
   \end{bmatrix}
   \begin{bmatrix}
   dy \\
   dr \\
   dp
   \end{bmatrix}
   =
   \begin{bmatrix}
   (1 - c_1)dg \\
   0 \\
   0
   \end{bmatrix}
   \]

   \[
   J = -(i' + c_2) + (f_n/f_{nn})[(1 - c_1)m_r + (i' + c_2)m_y] > 0
   \]

   Even though the budget deficit doesn’t change, an equal increase in $g$ and $T$ causes output and the interest rate to rise. It should be clear why this is the case: ceteris paribus, a unit increase in $g$ causes demand to rise by a unit; but a unit increase in $T$ causes demand to fall by less than a unit. Once we know that demand increases, the mechanism is also clear: demand raises $p$, which reduces real wages, increases employment, and increases output. Output will rise less than one-for-one with balanced budget spending, in equilibrium. As output and the price level rise, money demand will exceed money supply. The interest must therefore adjust upwards to maintain equilibrium in this market.

   b. Sticky-price model. I have used the block recursive structure of this model to simplify.

   \[
   \frac{M}{p} = m(r, y) \\
   y = c(y - T, r) + i(r) + g
   \]

   \[
   \begin{bmatrix}
   m_y \\
   1 - c_1 \\
   -c_2 + i'
   \end{bmatrix}
   \begin{bmatrix}
   m_r \\
   1 - c_1 \\
   -(c_2 + i')
   \end{bmatrix}
   \begin{bmatrix}
   dy \\
   dr \\
   dp
   \end{bmatrix}
   =
   \begin{bmatrix}
   0 \\
   (1 - c_1)dg \\
   0
   \end{bmatrix}
   \]

   \[
   J = m_y(c_2 + i') - (1 - c_1)m_r > 0
   \]

   \[
   \frac{\partial y}{\partial g} = \begin{bmatrix}
   0 \\
   0 \\
   0
   \end{bmatrix}
   \frac{1}{J} = \frac{(1 - c_1)m_r}{J} > 0
   \]

   \[
   \frac{\partial r}{\partial g} = \begin{bmatrix}
   m_y \\
   1 - c_1 \\
   -(c_2 + i')
   \end{bmatrix}
   \frac{1}{J} = \frac{(1 - c_1)m_y}{J} > 0
   \]

   As before, the fiscal policy experiment raises demand, which in this model is fully accommodated by supply. Because $M$ and $p$ are exogenous, and because the demand for money rises with income, the interest rate must rise by enough to offset the income effect, so that money demand does not change.
2. Endogenous expectations
   a. aggregate supply

\[
d y = f_N \left[ \frac{N'(1-h'(p))}{1-N'f_{NN}} \right] dp + \left[ \frac{f_N}{1-N'f_{NN}} \right] dz
\]

\[
= a_1 dp + a_2 dz
\]

As long as \( 0 < h'(p) < 1 \), an increase in \( p \) reduces the real wage more than it reduces the expected real wage, so (equilibrium) employment rises, and therefore so does output supplied.

b. Changes in \( g \). Using (a), the model is

\[
\begin{pmatrix}
1 & 0 & -a_1 \\
1 & -y_1 & 0 \\
m_y & m_r & 1
\end{pmatrix} \begin{pmatrix}
\frac{dy}{dr} \\
\frac{dy}{dp} \\
\frac{dr}{dm}
\end{pmatrix} = \begin{pmatrix}
a_x dz \\
y_2 dg \\
m_r
\end{pmatrix}
\]

\[
J = -[y_1 + a_1(m_r + m_y y_1)] > 0
\]

\[
\frac{\partial y}{\partial g} = \frac{-y_2 m_r a_1}{J} > 0
\]

\[
\frac{\partial r}{\partial g} = \frac{y_2(1 + m_y a_1)}{J} > 0
\]

\[
\frac{\partial p}{\partial g} = \frac{-y_2 m_r}{J} > 0
\]

A potential explanation: when \( g \) rises, aggregate demand rises, which puts upward pressure on the interest rate. The higher interest rate reduces money demand, which tends to increase \( p \) as households try to get rid of money. In general, the price level rises higher than the expected price; thus, the real wage will fall while the expected real wage will rise. Both firms and households are therefore happy to increase employment. The increase in \( N \) leads to more output supplied, which accommodates the initial increase in aggregate demand.

c. In the classical model, \( a_1 = 0 \). Then

\[
\frac{\partial r}{\partial g} \bigg|_{a_1=0} = \frac{-y_2}{y_1} > 0.
\]

By manipulating the multiplier in part (b), we get

\[
\frac{\partial r}{\partial g} = \frac{y_2(1 + m_y a_1)}{J}
\]

\[
= \frac{-y_2}{y_1} \left[ \frac{1 + m_y a_1}{(1 + m_y a_1) + \frac{a_1 m_r}{y_1}} \right]
\]

Since the bracketed term is clearly less than one, the former multiplier is bigger than the latter. In the classical model, the interest rate must rise a lot to ensure that demand does not rise at all.

d. Changes in \( z \):

\[
\frac{\partial y}{\partial z} = \frac{-a_2 y_1}{J} > 0
\]

\[
\frac{\partial r}{\partial z} = \frac{-a_2}{J} < 0
\]

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