Instructions: If you do not use a word processor (e.g. Word or TeX), please write as neatly and legibly as possible. You are free to discuss these problems with classmates, but what you turn in must reflect independent work.

1. The model below adds a flat rate income tax to a simple linear macroeconomic model, and requires that the government deficit be financed by money creation. The model is

\[ y_t = -a_1 r_t + g_t \]
\[ m_t = b_1 y_t - b_2 r_t \]
\[ \tau_t = \mu_0 + \mu_1 y_t \]
\[ m_{t+1} = m_t + g_t - \tau_t, \]

where \( y \) is output, \( r \) is the interest rate, \( g \) is government spending, \( m \) is the nominal money stock, \( \tau \) are tax revenues collected by the government, and all coefficients are positive. The price level \( p \) is exogenous by assumption and is therefore ignored in this problem. The first equation is a simplified aggregate demand relationship (the “IS” curve), the second shows the money market equilibrium condition, the third defines the income tax (\( \mu_0 \) is a lump sum amount, while \( \mu_1 \) is the marginal income tax rate), and the fourth requires the Fed to issue money (next period) if there is a budget deficit (there are no bonds). Assume that \( y_t, r_t, \tau_t, \) and \( m_{t+1} \) are the endogenous variables. The money stock is exogenous at a point in time, but endogenous over time.

a. Find the dynamic multiplier \( \frac{\partial \tau_t}{\partial g_t} \), for all \( k \).

b. What happens to tax revenues in the long-run if the government increases \( g \) once and for all (i.e. permanently)? Explain.

c. What happens to tax revenues in the long-run if lump-sum taxes (\( \mu_0 \)) are increased once and for all (assuming \( g \) is exogenous)? Explain.

d. Suppose we want to “endogenize” the price level? How would you alter the model to allow the price level to be endogenous? (You don’t need to solve this model; just indicate how you would set it up.)

2. For the household’s two-period, dynamic consumption/leisure choice problem, assume that \( u(c, n) = \ln(c) + \ln(e - n) \), where \( c \) is consumption, \( n \) is hours of labor supplied, and \( e \) is an exogenous endowment of “time.” Assume that the interest rate \( r \) is exogenous, and that \( \beta = \frac{1}{1 + r} \).

a. Using the first order conditions for this problem, given in the notes, compute the optimal time paths for consumption and labor supply.

b. Compute the effect of a permanent change in wages (i.e. \( dw_0 = dw_1 \)) on \( c_0, c_1, n_0 \) and \( n_1 \). Interpret the results.

c. Repeat this exercise for a temporary change in wages (i.e. \( dw_0 = -dw_1 \)).

d. What happens to the optimal consumption and labor paths if the household is endowed with more time (i.e. \( e \) goes up)?