1. Consider the simple model of optimal intertemporal consumption developed in class (where
the interest rate and the time path of income are exogenous, and all variables are defined as
in class):

\[
\text{Max } \sum_{t=0}^{\infty} \beta^t u(c_t)
\]

subject to \( A_{t+1} = (1 + r)(A_t + y_t - c_t), \forall t \)

a. Provide a brief economic interpretation of the transversality condition implied by this
problem: \( \lim_{T \to \infty} (1 + r)^{-T} A_T = 0 \).

b. What does this model predict will happen to the optimal consumption path when i) the
real interest rate falls? ii) when current income increases temporarily? Explain.

c. Suppose that the government imposes a proportional tax on consumption in all periods.
Rewrite the household’s intertemporal budget constraint to reflect the tax. With respect
to the two period graph of this problem, how do you think this tax will affect the slope of
the intertemporal budget line?

2. The model in problem 1 can be generalized to endogenize income by incorporating optimal
labor supply, where the wage path is taken as given. For the following questions, you can
assume the two period model discussed in class.

a. In general, and holding the time path of consumption fixed, what is likely to happen to
current and future labor supply if the real interest rate rises? Explain.

b. Again, suppose that the optimal consumption path is fixed, and that the government
imposes a proportional tax on labor income. Explain the likely effects on the household’s
optimal time path for leisure if the tax is temporary (i.e. imposed only during the current
period); and if the tax is permanent (i.e. the tax is imposed in both the current and future
periods).

3. Provide a brief description of the basic Real Business Cycle model discussed in class and
Lucas’s model of the business cycle that depends on imperfect information. How are these two
models the same? How are they different?

4. In the model of optimal investment with adjustment costs, the shadow value \( q \) plays an
important role. In economic terms, interpret \( q \) and explain what role it plays in determining
the dynamics of optimal investment. Also, explain what happens to \( q \) in the short-run and
long-run if the real interest rate rises (starting from an initial steady-state).
5. The questions below pertain to an economy in the long-run, when the aggregate stock of capital is endogenous.
   a. Suppose a politician calls for policies to increase the rate of saving in the economy to promote economic growth. According to the neoclassical (Solow) model of economic growth, will such policies be likely to achieve the objective of sustained growth rates for per capital output? Why or why not?
   b. Within the framework of Solow’s model of growth, use a graph to explain the effects on per capita output and capital of a technological innovation that reduces the rate of depreciation ($\delta$). (Assume that the growth rate of technology ($g$) is zero.)
   c. Use Ramsey’s model of optimal growth to describe the dynamics of the real interest rate if the economy begins on the saddle-path to the left of the steady-state capital stock (i.e. where the initial capital stock is less than its steady state value). Be sure to explain the ultimate determinant of the real rate in the long-run steady state.

Part II

6. The on-going conflict in Iraq has, without question, increased overall federal government spending, as well as the federal budget deficit. A year after the war began, a NY Times article (4/30/04) claimed that this spending had significantly contributed to the rise in US real GDP over the first quarter of 2004.
   a. In the context of the static model of the macro economy discussed in class, under what conditions will the article’s claim be correct; i.e. that this (exogenous) increase in government spending, holding tax revenues fixed, caused an increase in aggregate output? Give a specific example of the mechanism by which this effect could have occurred.
   b. Under what conditions will the increase in government spending have no effect on real GDP? Explain.
   c. Suppose that Congress raises tax revenues to finance war expenditures. Given these expenditures, under what conditions will this tax increase per se, as opposed to borrowing, be irrelevant to the determination of aggregate output, consumption and investment?

7. Consider the following model of the macroeconomy:

\[ y_t = a_1 (p_t - p_t^e) \]  \hspace{1cm} (1)

\[ y_t = -b_1 p_t + b_2 m_t + b_3 g_t \]  \hspace{1cm} (2)

\[ p_t^e = p_{t-1} \]  \hspace{1cm} (3)

where $y$ is the natural log of aggregate output, $p$ is the natural log of the price level, $p^e$ is household’s expectation of the log price level, $g$ is the natural log of government expenditures, and $m$ is the natural log of the nominal stock of money. The first equation is the aggregate supply curve, the second is the aggregate demand curve, and the third models price expectations. The parameters $a_1, b_1, b_2$ and $b_3$ are positive. Assume that $y$ and $p$ are the endogenous variables in the model.

   a. Show that the equilibrium price level implied by the economic model follows a first-order difference equation.
   b. Recall that the solution to the difference equation $z_t = h_0 + h_1 z_{t-1} + h_2 x_t$ is $z_t = \frac{h_0}{1-h_1} + h_2 \sum_{i=0}^{\infty} h_i x_{t-i}$, for $|h_1| < 1$. Compute the dynamic multipliers $\frac{\partial y_{t+i}}{\partial g_t}$ and $\frac{\partial y_{t+i}}{\partial g_t}$ for all $i$.
   c. Compute the long-run effect of a permanent change in government spending on output and the price level (and show your work). Provide an economic interpretation of your result.
Answers

1. Intertemporal consumption.
   a. The present value of end of period wealth must approach zero (as long as the marginal utility of consumption is positive). This is a necessary condition for optimization. Otherwise, wealth is left over at the end, which could be consumed to increase utility.
   b. i) A fall in $r$ causes the budget line to rotate counterclockwise around the NB/NL point. If the HH is originally a lender, it will decrease future consumption unambiguously; the effect on current consumption will be ambiguous because the income and substitution effects work in opposite directions. If the HH is a borrower, current consumption will rise, since both effects work in the same direction; the effect on future consumption will be ambiguous. ii) A temporary increase in income, defined as a current increase offset by a decrease in future income, will have no or very little effect on both current and future income.
   c. Let $\theta$ be the proportional consumption tax rate. Then

\[ A_{t+1} = (1 + r)\left[A_t + y_t - (1 + \theta)c_t\right] \]

Because the tax is collected in both periods, the intertemporal tradeoff is not affected and the slope of the budget line will not change.

2. Labor supply
   a. An increase in the interest rate will cause the intertemporal budget line (reflecting the available tradeoff between current and future labor) to rotate clockwise. Therefore, it is likely that current labor will increase and future labor will decrease: the HH will choose to work more now (consume less leisure now), save more of the increased income to support the consumption path at the higher interest rate, and consume more leisure in the future, assuming the substitution effect outweighs the income effect. For a given consumption path, the higher rate increases the return to current labor, in terms of future labor.
   b. The proportional tax is tantamount to a decrease in wages. Thus, if the tax is permanent, it will not affect the intertemporal relative price of leisure – there will be little if any effect on leisure in either period (but consumption would be affected). On the other hand, if the tax is imposed on current labor but is removed next period, current leisure would most likely rise (as current labor supply falls because of the tax). The HH will reduce future labor to work more during the period of low taxes (again, assuming predominance of the substitution effect).

3. RBC model – The economy consists of a representative household/consumer and firm, who interact in competitive markets for produced goods, labor and capital. HH’s solve for their optimal consumption path given an intertemporal utility function subject to an asset accumulation constraint. HH saves by accumulating capital, which it rents to firms for use in production. Prices in all markets are flexible – they clear all markets so they are always in equilibrium. Real technology shocks drive aggregate fluctuations in this model, which are optimal responses to these shocks. Fluctuations are therefore not due to deviations around full-employment output. The dynamics of the business cycle come from the optimal consumption smoothing behavior of the hh.

Lucas assumes the economy consists of disparate markets for goods, separated by imperfect information flows. While producer/households observe their individual market prices, they can’t observe the overall price level when making production decisions. Thus, they are confused in
general about the source of shocks in the economy. In this world, a purely nominal, aggregate shock (to the money supply) will cause an increase in aggregate output, since individual producers believe – to some extent – that the shock is due to real factors (relative demand). The model can therefore explain money as a causal source of the business cycle, even when markets clear and expectations are rational. Note also that the model predicts that the sensitivity of output to aggregate demand/money shocks is inversely related to the variance of these shocks. Both models are similar in that there are no market/price rigidities as in Keynesian models. They differ in terms of the types of sources of fluctuations.

4. \( q \) is the marginal value to the firm of an increase in the stock of capital; it is the present value of future (net) returns to capital. Present-value-maximizing firms will have an incentive to invest (accumulate capital) when \( q \) exceeds 1 plus the marginal adjustment cost of investment. When \( q = 1 \), optimal net investment is zero, and the stock of capital will be optimized.

Start in the steady-state, where \( q = 1 \) and capital is stationary. An increase in \( r \) shifts the \( \Delta q = 0 \) locus to the left, and the saddlepath down. Because capital is initially fixed, \( q \) must fall immediately to the new saddlepath, or the system would not converge. Because \( q \) then falls below one, capital will fall gradually over time. But since the capital stock is ‘too large’ and the return to capital is thus ‘too small’, \( q \) will have to rise over time to ensure that the user cost of capital is small enough during the transition (remember that capital gains from \( q \) reduce the user cost).

5. Growth
   a. In the Solow model, an increase in the rate of saving will lead to a long-run increase in output per person, but will not affect the long-run growth rate of the economy. Because of diminishing returns to capital, the sustained rate of growth of output per person will be determined solely by the growth rate of technology.
   b. When \( \delta \) falls, the capital deterioration line (ray from the origin) rotates clockwise. At the initial stock of capital, accumulation through saving becomes greater than the reduction of the intensive form of capital, so the capital stock gradually rises during the transition. As capital rises, output rises through the production function. Ultimately, capital and output (per effective labor unit) will increase to higher levels and will converge to a new steady state. In this steady-state, output and capital will grow at rate \( n \) (the growth rate of labor).
   c. At the initial level of capital, future consumption will rise relative to current consumption because capital is low and the return to capital is high (thus satisfying the Euler equation). That is, the interest rate is high relative to household’s rate of time preference. At the same time, capital will rise because consumption is lower (saving is higher) than is needed to maintain the capital stock, so that the interest rate will be falling over during the transition. Ultimately, capital will rise, and its return will thus fall, until the return to capital (its marginal product) equals the household’s rate of time preference. In the steady-state, as long as the rate of time preference is positive, capital stock will be below the golden-rule level of capital.

6. Static model.
   a. We would have to assume some form of market rigidity or imperfect information. If, say, nominal wages are exogenous, then the increase in government spending would lead to higher interest rates to reduce aggregate spending, which in turn would reduce money demand and put upward pressure on prices to ensure that the real stock of money satisfies
the portfolio balance equation. Higher goods prices lead to lower real wages thereby increasing employment (along the labor demand curve). Finally, the additional employment would lead to an increase in output to accommodate the increase in demand.

b. If prices are flexible and all markets clear and there is perfect information about these prices, then the change in government spending would not affect the level of real GDP (though it would affect the composition of total output). The increase in the interest rate would be large enough to fully crowd out private spending. At the same time, as rates rise and money demand falls, the price level rises, but this is matched by proportionate changes in wages, so that real wages remain constant and there is no incentive for firms to increase output.

c. Under the conditions that lead to Ricardian Equivalence, households will simply pay the higher taxes by reducing saving, therefore leaving national saving unchanged. There will therefore be no change in the economy.

7. Dynamic model

a.

\[ p_t = \frac{a_1}{a_1 + b_1} p_{t-1} + \frac{b_1}{a_1 + b_1} m_t + \frac{b_3}{a_1 + b_1} g_t \]

b.

\[ \frac{\partial p_t}{\partial g_{t-i}} = \frac{b_3}{a_1 + b_1} \left( \frac{a_1}{a_1 + b_1} \right)^i \]

\[ \frac{\partial y_t}{\partial g_t} = b_3 - b_3 \frac{b_1}{a_1 + b_1} \]

\[ \frac{\partial y_t}{\partial g_{t-i}} = -b_1 \left( \frac{b_3}{a_1 + b_1} \right) \left( \frac{a_1}{a_1 + b_1} \right)^i, \quad i = 1, \ldots \]

c. The sum of the output multipliers gives the long run effect of a permanent change in \( g \). This sum is easily seen to be zero. In the short run, government spending affects output; but the effect fades over time as expectation of price increases ‘catch up’ to the actual price level.