1. Consider the following macroeconomic model. All variables and notation are as defined in class.

\[
\frac{w}{p} = f_N(N) \quad (1)
\]

\[
N = N\left(\frac{w}{p}\right), \quad N' > 0 \quad (2)
\]

\[
y = f(N), \quad f'(N) > 0, \quad f''(N) < 0 \quad (3)
\]

\[
y = y(r), \quad y' < 0 \quad (4)
\]

\[
\frac{M}{p} = m(r, y), \quad m_1 < 0, m_2 > 0 \quad (5)
\]

(1) is the labor demand curve, (2) is labor supply based on household’s expected price level, (3) is the production function, (4) is the aggregate demand identity (the IS curve), and (5) is the money market equilibrium condition. \(M\) is exogenous. Unless stated otherwise, assume all prices are flexible (i.e. endogenous).

For each of the cases below, describe the effect of an exogenous increase in the money supply on output and the interest rate, and the mechanism by which this effect occurs. The good news is that you need not compute any multipliers; I want an economic explanation in words of how money is transmitted through the economy in each case.

a. \(p_e = g(p)\) where \(0 < g'(p) < 1\)

b. \(p_e = p\)

c. \(p_e = p\), \(w\) is exogenous, and employment equals the demand for labor.

2. A consumer lives two periods (periods 0 and 1) with perfect foresight, and determines her lifetime consumption plan by solving the constrained optimization problem below. In particular, she maximizes

\[
V_0 = ln(c_0) + \beta ln(c_1)
\]

subject to her lifetime budget constraint

\[
c_0 + \frac{c_1}{1 + r} = y_0 + \frac{y_1}{1 + r}.
\]

Assume that the interest rate \(r\) is taken as given, as is her current and future income flows, \(y_0\) and \(y_1\), respectively. Also, \(0 < \beta < 1\), and the interest rate and income flows are positive.
a. Solve this problem for the consumer’s optimal time path, \( c_0 \) and \( c_1 \). Your answer should be expressions showing optimal \( c_0 \) and \( c_1 \) as functions of the exogenous variables.

b. Use the solution in (a) to show what the model predicts will happen to her current consumption (\( c_0 \)) when the interest rises. Explain.

c. What happens to current consumption when there is a ‘permanent’ increase in her income? Explain.

3. Briefly identify, define or compare (in one or two sentences) the following concepts.
   a. Monetary neutrality
   b. The liquidity effect
   c. The transversality condition
   d. Rational expectations versus adaptive expectations
   e. The Ricardian Equivalence proposition
   f. An efficiency wage

4. Assume that investment is determined by the q-theory model discussed in class.
   a. Draw a phase diagram of the model assuming no investment tax credit. Be sure to carefully label the diagram. Point out the steady-state of the system, and explain why it is a steady-state. Describe the potential dynamics of \( q \) and \( K \) outside the steady-state.
   b. Consider a government policy that returns \( \theta \) dollars to the firm for each dollar invested, i.e. an investment tax credit. Assume that such a policy is announced and implemented on the same day. Using a separate phase diagram and starting from an initial steady-state, explain the dynamics of the shadow price (\( q \)) and the capital stock in response to this policy.

5. Growth models.
   a. Some economists believe that the impressive growth in output per capita in many of the newly industrialized economies (e.g. Taiwan) is due solely to capital accumulation. If true, what does Solow’s model predict about the ability of these economies to sustain such high rates of growth? Why?
   b. True or false: in Solow’s model, a reduction in the proportion of income saved by households reduces the growth rate of per capita income in the steady-state. Explain.
   c. Define the “golden rule” level of the capital stock. Under the standard assumptions of Ramsey’s growth model, why is this level of the capital stock not optimal?
   d. What ultimately determines the long-run interest rate in Ramsey’s model (assuming that there is no technological improvement; i.e. \( g = 0 \))? Briefly explain the economic behavior underlying your answer.
Answers

1. A static macro model and monetary transmission
   a. This condition implies that households have imperfect information about the actual price level, but firms have full information. Suppose the money supply increases. Initially, through the money market, the incipient excess money supply causes $p$ to rise. As $p$ rises, the actual real wage falls, but the household’s projected real wage rises. Because both labor demand and labor supply increase, employment rises, and output will rise via the production function. The interest rate falls to ensure that demand increases along with supply. Alternately, the rise in $M$ causes $r$ to fall, which reduces aggregate demand relative to supply. $p$ then rises, and the rest of the story is the same.
   b. This is the case of perfect price flexibility in all markets and perfect information – the classical model. As $M$ rises, both $p$ and $w$ rise proportionately, so that real wages don’t change. There is therefore no incentive for change in the labor markets; employment will not change. Since inputs remain the same, so does output. Neither output nor the interest rate will change – money is neutral.
   c. As money rises, the price level will rise. Since the nominal wage is exogenous, it won’t adjust. Thus, the real wage falls, firms have an incentive to hire more workers, which they do. Output rises by the production function. The interest rate falls to increase aggregate demand to accommodate the supply effect.

2. Intertemporal consumption.
   a. Optimal consumption paths:
      \[
      c_0 = \frac{1}{1 + \beta} (y_0 + \frac{y_1}{1 + r})
      \]
      \[
      c_1 = \beta (1 + r) \frac{y_0 + y_1}{1 + \beta (1 + r)}
      \]
   b. The interest rate will have a negative effect on $c_0$. The substitution effect will outweigh the income effect (if they move in opposite directions) no matter what.
      \[
      \frac{\partial c_0}{\partial r} = -\left( \frac{1}{1 + \beta} \right) \frac{y_1}{(1 + r)^2} < 0
      \]
   c. Permanent income
      \[
      \frac{\partial c_0}{\partial y} = \frac{1}{1 + \beta} \frac{2 + r}{1 + r}
      \]

   a. Monetary neutrality: exogenous changes in money supply have no effect on real quantities and prices; changes in money cause proportional changes in all nominal magnitudes.
   b. Liquidity effect: the negative effect on interest rates of an exogenous change in the money supply as bonds are substituted for money.
c. Transversality condition: end-period optimal condition of a dynamic optimization problem. For example, in the consumer problem, the condition implies that it is optimal to have no wealth left over after the final period.

d. Expectations: Adaptive expectations is naive projection based only on the past; rational expectations assume agents know the true model determining the variable and do not make systematic forecast errors.

e. Ricardian Equivalence: Government borrowing and taxation are the same, assuming government spending is fixed, if household realize that deficits are equivalent to future taxes.

f. A wage higher than the equilibrium wage that induces efficient work effort from labor suppliers, perhaps due to asymmetric information. Based on the general idea that wages can be benefits to firms, not just costs.

4. q-theory of investment
a. $q = 1$ defines the $I = 0$ locus; if $q > 1$, $I > 0$. The locus of $q, K$ such that $\dot{q} = 0$ is downward sloping; it is based on the Euler equation that sets the return to capital equal to its user cost. If $K$ exceeds this locus for any given $q$, the return to capital on the margin is low and $q$ must rise in the future to create capital gains which lower user cost. The intersection of these two loci determine the steady-state of the system: neither $q$ nor $K$ will move from this point given the loci. The system is clearly unstable for points above $I = 0$ and to the right of $\dot{q} = 0$, as well as below $I = 0$ and to the left of $\dot{q} = 0$. The saddlepath is downward sloping.

b. The announced ITC will shift the investment locus down to $1 - \theta$; in effect, the relative price of capital falls from 1 to $1 - \theta$. The saddlepath will also fall. Upon announcement and implementation, $q$ will fall immediately to the new saddlepath. Because $q$ remains above the zero-investment locus, investment will rise, increasing the capital stock. But the point lies to the left of the $\dot{q} = 0$ locus, so $q$ falls as the steady-state is approached.

5. Growth models.

a. They would likely not sustain such growth because of diminishing returns to capital.

b. False. Such a reduction would reduce the steady-state level, but not the growth rate.

c. The golden rule level of capital is that level that maximizes per capita consumption. As long as the discount factor is less than one, this level of capital exceeds the optimal level. This is because achieving this level of capital would require too much sacrifice of current consumption, relative to households’ level of impatience.

d. The long-run interest rate equals the household’s rate of time preference, in the steady-state. Suppose that the interest rate, or return to capital, exceeds the rate of time preference (for example, suppose time preference rate $i$ falls, $\beta$ rises for a given capital stock). Then households will optimize in the short-run by reducing current consumption relative to future consumption; i.e. saving will rise and capital will accumulate. As the capital stock accumulates, the return to capital
falls due to diminishing returns. The capital stock will continue to grow until the steady-state condition $i = r$ is achieved.