1. Suppose that the economy solves the following dynamic optimization problem: maximize \( \sum_{t=0}^{\infty} \beta^t \ln(c_t) \), subject to \( y_t = c_t + k_{t+1} \) and \( y_t = \phi k_t^\alpha \), where \( c \) is consumption, \( y \) is output, and \( k \) is the capital stock, all per capita (assume the population is fixed). The first constraint restricts output to be totally used up for consumption and investment (since the rate of capital depreciation is assumed to be 100%); the second constraint is the production function where \( \phi \) and \( \alpha \) are parameters. In the analysis below, you can simply assume that the transversality condition must be satisfied along the optimal path.
   a. Write the Lagrangian function for this dynamic optimization problem.
   b. Write the two necessary conditions that define the optimal time path for consumption and capital. [Hint: solve for the first order conditions and eliminate the multiplier(s).]
   c. Solve for the optimal, steady-state capital stock (per capita).
   d. Solve for the optimal steady-state consumption (per capita).
   e. Using an appropriate graph, describe the transition to the new steady-state after the utility maximizer becomes “less patient.” Frame your description in terms of the optimal behavior implied by the model.

2. Suppose that as a policy advisor, you are asked to explain the effects of a proposed change in government spending on the macro economy. Write down a basic, static macro model that has the following characteristics: a) the endogenous variables are aggregate output, the interest rate, and the price level; and b) the exogenous variables are government spending and the money stock. Explain the equations you include in the model and any assumptions you make. Use the model to predict the effect of an increase in government spending on output, the price level and the interest rate; i.e. compute the appropriate reduced form multipliers. Explain the underlying economics implied by the multipliers. [Hint: note that you are free to make reasonable simplifying assumptions that make the analysis easier.]

3. Answer the following questions about the nature and role of the real interest rate in macro models.
   a. What is the liquidity effect? What conditions are sufficient to generate such an effect in the static macro models developed in class? Why is the liquidity effect important in these models?
   b. How does the optimal consumption path of a rational consumer, who maximizes his or her lifetime utility function subject to an intertemporal budget constraint, respond to an increase in the real interest rate? Use an appropriate graph to help with your explanation.
   c. According to the q-theory of investment developed in class, how does a value-maximizing firm, facing quadratic adjustment costs of investment, respond to a decrease in the real interest rate with respect to its investment decision? Use an appropriate graph to help with your explanation.
   d. Is your answer in (c) compatible with the implications of optimal growth theory that a higher interest rate provides an incentive for greater saving, and thus greater capital accumulation? Explain.
   e. What ultimately determines the steady-state interest rate in model of optimal growth discussed in class? Explain.
4. Provide a concise answer to each of the questions that follow.
   a. Distinguish between ‘adaptive expectations’ and ‘Rational Expectations’. In the context of the simple aggregate supply-aggregate demand model discussed in class, what are the implications of each of the assumptions above for the effectiveness of monetary policy?
   b. How is an aggregate, nominal shock to the money stock ‘transmitted’ to the real economy in Lucas’s model of imperfect information (i.e. how does money affect aggregate output in the ‘island’ model)?
   c. What is an ‘efficiency wage’, and what do efficiency wage models typically imply about unemployment?
   d. Give a brief theoretical explanation for why we might expect a temporary change in real wages to have a larger effect on labor supply than a permanent change in real wages.
   e. Suppose the saving rate in a country increases. According to Solow’s neoclassical growth model, what effect does this have on the country’s long-run growth rate of output per person.
Suggested answers

1. The model is one of optimal growth, in which we can think of a market economy as being represented by a central planner. Note that the constraints hold for each period, so there is a multiplier for each constraint.

   a. \[
   \sum_{t=0}^{\infty} \beta^t \ln(c_t) = \sum_{t=0}^{\infty} \lambda_t (\phi k_t^\alpha - c_t - k_{t+1})
   \]

   b. Take the derivative of the Lagrangean with respect to \(c_t\) and \(k_{t+1}\) (for all \(t\)), then eliminate the multipliers to get the Euler equation:

   \[
   \frac{\beta c_t}{c_{t+1}} = \frac{1}{\phi \alpha k_{t+1}^{\alpha - 1}}
   \]

   The second necessary condition is the constraint itself:

   \[
   \phi k_t^\alpha - c_t - k_{t+1} = 0
   \]

   c. Assume \(c\) is constant in the Euler equation. This yields a unique value for \(k\):

   \[
   k^* = (\phi \alpha \beta)^{\frac{1}{1-\alpha}}
   \]

   d. Use the value in part (c) in the constraint, assuming \(k_t = k_{t+1}\):

   \[
   c^* = \phi k^* \alpha - k^*
   \]

   b. As the household becomes less patient, the steady-state capital shifts to the left. Since capital is fixed in the short-run, consumption jumps up immediately to a higher level, to the new saddlepath. In the long-run, since capital accumulation slows, both consumption and capital will approach lower levels in the steady-state.

2. This question allows substantial freedom in setting up a model, so there is no single correct answer. I was looking for a model that is coherent (it can be appropriately solved) and sensible (the economics are consistent with what we discussed in class). My inclination if asked this question would be to write down the sticky-wage model, after collapsing the system into three equations in three unknowns \((y, r, P)\):

   \[
   y = y(P)
   \]

   \[
   y = c(y) + i(r) + g
   \]

   \[
   \frac{M}{P} = m(r, y).
   \]

   The first equation is aggregate supply, which subsumes the labor market and the production function (where I’ve ignored the capital stock for convenience). It is a function of the price level because of the assumption of sticky wages. The second equation is desired expenditures (the IS curve), while the third is the money market equilibrium condition (LM curve). Obtaining the multipliers with respect to \(g\) is a simple matter; making the usual assumptions about elasticities, you would see that an increase in \(g\) increases \(y, r\) and \(P\).

3. Interest rate questions.
a. The liquidity effect is the negative response of interest rates to an exogenous change in the nominal stock of money. It will not arise the simple flexible price model, but will in models with market or information rigidities. It is important in these models because the interest rate response plays an essential role in transmitting nominal money shocks to the real economy.

b. An increase in $r$ rotates the consumers intertemporal budget line clockwise around the no borrowing/no lending point. However, his or her response depends on whether he or she is a lender or a borrower. If the former, future consumption will unambiguously rise, since the income and substitution effects work in the same direction. However, the response of current consumption is ambiguous, because these effects work in opposite directions. For a borrower, current consumption will definitely decrease as $r$ goes up; future consumption is ambiguous.

c. A decrease in $r$ shifts the $\Delta q$ locus to the right, and the saddlepath up. Since capital is initially fixed, $q$, the shadow price of capital will rise above one to the new saddlepath, causing investment to be positive and capital to accumulate. At the same time, $q$ will fall. The saddlepath is justified to satisfy the transversality condition. Finally, $q$ returns to 1, investment will fall to zero, and the capital stock is back in steady-state.

d. The optimal growth theory discussed in class assumes that the capital stock is always in long-run equilibrium; i.e. $q = 1$ always. When this is true, the interest rate is the same as the marginal product of capital; in the short-run investment model, these two concepts can differ, which generates investment. Thus, they are consistent.

e. In the steady-state, the capital stock will adjust until the marginal product of capital – the interest rate – equals the rate of time preference of households (assuming no technology growth).

4. Shorter questions.

a. Adaptive expectations – naive model of expectations, assuming agents use only past behavior (e.g. a distributed lag) of the variable being forecast. On the other hand, rational expectations assumes that agents are correct, on average, in making forecasts. They do this by using the objective expected value based on the model in which the forecasts play a role. Under adaptive expectations, feedback rules for monetary policy are effective; i.e. they can help the central bank achieve a real income target. Under RE, such rules are ineffective since agents can presumably anticipate the actions of the central bank and will adjust their behavior.

b. Lucas’s model is based on the assumption that households/producers have imperfect information about the aggregate economy; before economic decisions are made, they observe only their particular market price. Suppose the central bank injects money into the economy, which is not immediately observed by agents. Local producers observe their prices rising, but to some extent (depending on the relative variation of relative and aggregate shocks) they perceive these purely nominal price rise as a relative price rise. They then rationally respond by increasing output. Since all producers are the same, aggregate output will respond positively to this aggregate, nominal money supply shock.

c. Efficiency wages arise when wages are assumed to have a benefit to firms, as well as a cost. For example, if work effort positively depends on the wage, efficiency wages might arise as firms choose an optimal wage – the wages that maximizes benefit per dollar paid to workers. Efficiency wages can lead to unemployment in equilibrium.

d. A temporary change in real wages affects the intertemporal tradeoff of leisure for an lifetime utility maximizer. A permanent change in the does not affect this tradeoff. It will
be optimal for the agent to substitute leisure/labor over time when wages are temporarily higher or lower.
e. The saving rate will not affect the steady-state growth rate of output per capita, since this rate is uniquely determined by the rate of technology growth. An increase in the saving rate will raise only the steady-state level of output.