1. Suppose that households increase labor supply in response to a rise in the real interest rate. Assume that all the other assumptions of the flexible price model hold.
   a. Compute the reduced form multipliers $\frac{\partial y}{\partial g}$ and $\frac{\partial r}{\partial g}$ and provide an explanation of these multipliers.

2. Consider an exogenous decrease in government spending that is completely matched by a decrease in tax revenues (i.e., $dg = d\tau$). Predict the effect of this hypothesized fiscal policy action on output, employment, and the nominal interest rate using a) the “sticky-wage” model and b) the “sticky-price” model as developed in class. For each model, you may use the following simplified expression for aggregate demand: $y = c(y - \tau, r) + i(r) + g$, where $i'(r) < 0$. Briefly explain and interpret these multipliers in terms of the implied economic behavior.

3. The static classical model discussed in class is consistent with Fisher’s hypothesis in that the equilibrium response of the real interest rate to an exogenous change in expected inflation is zero. Compute the reduced form multiplier $\frac{\partial (R - \pi)}{\partial \pi}$ implied by a) the rigid nominal wage and b) the imperfect information models discussed in class. Discuss the implications of these multipliers for Fisher’s hypothesis in each case.

4. Suppose we believe that crude oil is an input into the aggregate production process, along with capital and labor. Explain how you would extend the basic macroeconomic framework developed in class to account for this revised production process. You need not solve the model, but explain any equations you might add, the economic behavior underlying those equations, and so on.

5. Compute the reduced form multipliers for output and price in response to an exogenous change in the foreign interest rate for both the fixed exchange rate and flexible exchange rate regimes. Explain the underlying economic behavior and equilibrium adjustment implicit in these multipliers.
1. Labor supply and the interest rate.

a. The only change that needs to be made is to generalize the labor supply function as
\[ n = n(W, R), \]
where \( n_1 > 0 \) and \( n_2 > 0 \). The flexible-price (classical) system becomes
(after setting changes in all exogenous variables but \( g \) to zero, noting block recursive
structure in the first four equations, and assuming \( f_{kn} = 0 \)):\[
\begin{bmatrix}
0 & -f_{nn} & 1 & 0 \\
0 & 1 & -n_1 & -n_2 \\
1 & -f_n & 0 & 0 \\
(1-c_1) & 0 & 0 & i' - c_2
\end{bmatrix}
\begin{bmatrix}
dy \\
dn \\
dW \\
dR
\end{bmatrix}
= \begin{bmatrix}
0 \\
0 \\
g \\
g
\end{bmatrix}
\]
The Jacobian of the system (i.e. the determinant of the 4 \times 4 coefficient matrix) is
\[ J = -(1-c_1)f_n n_2 + f_{nn} n_1 (i' - c_2) - (i' - c_2) < 0. \]
Using Cramer’s Rule:
\[ \frac{\partial y}{\partial g} = \frac{-f_n n_2}{J} > 0 \]
\[ \frac{\partial R}{\partial g} = \frac{f_{nn} n_1 - 1}{J} > 0 \]
The multipliers show that both income and the interest rate rise with the increase in
government spending. It is easy to verify that, when \( n_2 = 0 \) (as in the original flex-price
model discussed in class), output does not change, and the change in the interest rate is
larger than when \( n_2 \) is positive. Here’s what’s going on: an increase in \( g \) leads to excess
demand for goods (same thing as excess demand to borrow since labor market clears,
according to Walras’s Law), which drives the interest rate up. Labor supply increases
(shifts to the right) due to \( n_2 > 0 \), driving the equilibrium real wage down and equilibrium
employment up. By the production function, output supply increases to accommodate
the increased demand for output. Because output rises, the interest rate need not rise as
high as it would in the original case, since demand need not fall by as much. Alternatively,
the increased income generates additional saving to accommodate loanable funds demand
at a lower interest rate. Note that the model is still classical in the sense that there is no
“excess” employment; the labor market still clears. But here, fiscal policy can have real
effects (on output) because it can affect labor supply through the interest rate.

b. Even though the system in this generalized model is no longer 3 \times 3 block exogenous, it
is 4 \times 4 block exogenous, meaning that money has no effect on the first four endogenous
variables. It effects only nominal prices and magnitudes, not real, so monetary neutrality
still holds.

2. Balanced budget multipliers. In each of the answers, I assumed that \( M = w = p = 1 \) as initial
equilibrium values for convenience. Don’t forget to set \( dg = d\tau \).

a. Sticky-wage model.\[
\begin{bmatrix}
1 & 0 & \frac{f_n}{f_{nn}} \\
(1-c_1) & -(i' + c_2) & 0 \\
m_y & m_r & 1
\end{bmatrix}
\begin{bmatrix}
dy \\
d\tau \\
dp
\end{bmatrix}
= \begin{bmatrix}
0 \\
0 \\
(1-c_1)dg
\end{bmatrix}
\]
\[ J = -(i' + c_2) + (f_n/f_{nn})[(1 - c_1)m_r + (i' + c_2)m_y] > 0 \]

\[
\frac{\partial y}{\partial g} = \begin{vmatrix} 0 & 0 & \frac{f_n}{f_{nn}} \\ (1 - c_1) & -(i' + c_2) & 0 \\ 0 & m_r & 1 \end{vmatrix} \frac{1}{J} = \frac{(1 - c_1)m_r(f_n/f_{nn})}{J} > 0
\]

\[
\frac{\partial r}{\partial g} = \begin{vmatrix} 1 & 0 & \frac{f_n}{f_{nn}} \\ (1 - c_1) & - (c_2 + i') & 0 \\ m_y & 1 \end{vmatrix} \frac{1}{J} = \frac{(1 - c_1) - m_y(f_n/f_{nn})}{J} > 0
\]

The decrease in \( g \) decreases demand, while the decrease in \( \tau \) increases demand; however, the former outweighs the latter because \( c_1 < 1 \). Thus, the decrease in spending causes \( P \) to fall, real wages to rise, and thus output to fall under this model.

b. Sticky-price model. I have used the block recursive structure of this model to simplify. I have also explicitly included consumption and investment to be clear about the role of taxes.

\[
\frac{M}{p} = m(r, y)
\]

\[ y = c(y - \tau, r) + i(r) + g \]

\[
\begin{bmatrix} m_y & m_r \\ 1 - c_1 & -(c_2 + i') \end{bmatrix} \begin{bmatrix} dy \\ dr \end{bmatrix} = \begin{bmatrix} 0 \\ (1 - c_1)dg \end{bmatrix}
\]

\[ J = -m_y(c_2 + i') - (1 - c_1)m_r > 0 \]

\[
\frac{\partial y}{\partial g} = \begin{vmatrix} 0 & m_r \\ 1 - c_1 & -(c_2 + i') \end{vmatrix} \frac{1}{J} = \frac{(1 - c_1)m_r}{J} > 0
\]

\[
\frac{\partial r}{\partial g} = \begin{vmatrix} m_y & 0 \\ (1 - c_1) & (1 - c_1) \end{vmatrix} \frac{1}{J} = \frac{(1 - c_1)m_y}{J} > 0
\]

3. Fisher’s hypothesis

a. Sticky-wage model

\[
\begin{bmatrix} 1 \\ 1 - c_1 \\ m_y \end{bmatrix} \begin{bmatrix} 0 & a & \frac{M}{p^2} \\ (i' - c_2) & 0 & \frac{M}{p^2} \\ 0 & m_r & \frac{M}{p^2} \end{bmatrix} \begin{bmatrix} dy \\ dr \\ dp \end{bmatrix} = \begin{bmatrix} 0 \\ (i' - c_2)dp \end{bmatrix}
\]

where \( a = \frac{f_n w}{f_{nn} p^2} < 0 \).

\[ J = (i' - c_2)\frac{M}{p^2} + a[(1 - c_1)m_r - m_y(i' - c_2)] > 0 \]

\[ 0 < \frac{\partial r}{\partial \pi} = \frac{(i' - c_2)\frac{M}{p^2} - am_y(i' - c_2)}{J} < 1. \]

Therefore, the change in the real rate is negative. Fisher’s hypothesis does not hold. Note that if \( a = 0 \), we get the classical result.

b. Imperfect information/sticky-expectation model. For simplicity, assume \( p = g(p) = 1 \).

\[ 0 < \frac{\partial r}{\partial \pi} = \frac{y_1[m_y f_n N'w(g' - 1) + M(f_{nn} N' - 1)]}{y_1[m_y f_n N'w(g' - 1) + M(f_{nn} N' - 1)] + m_r f_n N'w(g' - 1)} < 1. \]
Again, since the real rate falls, the Fisher hypothesis does not hold. If $g' = 1$ (perfect information about prices), the Fisher hypothesis holds.

4. This question essentially asks you to consider an input into production in addition to capital and labor. This requires adding a new variable, the quantity of oil, to the production function. We would also have to add equations representing the supply and demand of this input (analogous to the labor market). The demand for oil would be derived from the production function (which would presumably be determined by the equality of the price of oil and its marginal productivity). The simplest way to think about oil supply is that it is imported from abroad, and the supply is exogenous. You might assume that oil is supplied perfectly elastically from abroad at a given price, which means that the oil price would be exogenous and quantity endogenous. Alternatively, you might assume that the quantity is exogenous, implying that oil price is endogenous. Of course, you could assume a more generalized supply of oil, where both price and quantity are endogenous. (A more difficult approach would be to divide the domestic economy into 2 firms, one producing final goods, the other the intermediate good oil.) The final model would consist of seven equations – labor supply, labor demand, the production function (with oil as an argument), oil supply, oil demand, the IS curve, and the LM curve – in seven unknowns – real wage, employment, output, oil price, oil production, interest rate and price level.

5. Here are my solutions for the reduced form multipliers, based on the model and notation as developed in class.

Fixed exchange rates

\[
\frac{\partial y}{\partial r^*} = -\frac{K'y'h_1s}{J} < 0
\]
\[
\frac{\partial P}{\partial r^*} = -\frac{K'h_1s}{J} < 0
\]

An increase in the foreign interest rate reduces net capital inflows and therefore tends to reduce $B$, the balance of payments. This has a contractionary effect on output because the fall in $B$ reduces the domestic money supply (unless $s = 0$). As nominal money falls, the price level will fall, reducing output supply through the first equation. Note that these multipliers are zero when $s = 0$; if the central bank perfectly sterilizes the effect of the foreign interest rate on domestic money, there is no way for this to affect the rest of the economy.

Flexible exchange rates

\[
\frac{\partial y}{\partial r^*} = \frac{K'y'm_hr_2}{J} > 0
\]
\[
\frac{\partial P}{\partial r^*} = \frac{K'm_hr_2}{J} > 0
\]

Under flexible exchange rates, the foreign interest rate has an expansionary effect on output and the price level. The mechanism is totally different here. Now, as the foreign interest rate rises, $B$ remains at zero as capital inflows fall. But the domestic interest rate will start to rise with the decrease in loanable funds. As $r$ rises, the demand for money will fall, causing the price level to rise so that real money falls to match the lower demand. (Note that a decrease in money demand is similar to an increase in money supply in this regard.) As $P$ rises, aggregate
supply increases by the first equation. Note that the higher interest rate will reduce aggregate spending by the second equation. Thus, it must be the case that the nominal exchange rate will rise by more than the price level so that the real exchange rate will rise to stimulate demand for domestic output (you can verify this by computing the exchange rate multiplier).