Economics 8040  
Problem set 2  
Lastrapes  
Fall 2006

1. The model below adds a flat rate income tax to a simple linear macroeconomic model where the price level is exogenous, and requires that the government deficit be financed by money creation. The model is

\[ y_t = -a_1 r_t + g_t \]
\[ m_t = b_1 y_t - b_2 r_t \]
\[ \tau_t = \mu_0 + \mu_1 y_t \]
\[ m_{t+1} = m_t + g_t - \tau_t, \]

where \( y \) is output, \( r \) is the nominal interest rate, \( g \) is government spending, \( m \) is the nominal money stock, \( \tau \) are tax revenues collected by the government, and all coefficients are positive. The price level \( p \) is exogenous by assumption and is therefore ignored in this problem. The first equation is a simplified aggregate demand relationship (the “IS” curve), the second shows the money market equilibrium condition, the third defines the income tax (\( \mu_0 \) is a lump sum amount, while \( \mu_1 \) is the marginal income tax rate), and the fourth requires the Fed to issue money (next period) if there is a budget deficit (there are no bonds). Assume that \( y_t, r_t, \tau_t, \) and \( m_{t+1} \) are the endogenous variables. The money stock is *exogenous* at a point in time, but *endogenous* over time.

a. Find the dynamic multiplier \( \frac{\partial \tau_t}{\partial g_{t-k}} \), for all \( k \).
b. What happens to tax revenues in the long-run if the government increases \( g \) once and for all (i.e. permanently)? Explain.
c. What happens to tax revenues in the long-run if lump-sum taxes (\( \mu_0 \)) are increased once and for all (assuming \( g \) is exogenous)? Explain.
d. Suppose we want to “endogenize” the price level? How would you alter the model to allow the price level to be endogenous? (You don’t need to solve this model; just indicate how you would set it up.)

2. Look back at the rational expectations model of the Phillips curve, discussed in class (pp. 32-33). In writing the reduced form for equilibrium output, we assumed that the current demand and supply shocks (\( \epsilon^d_t \) and \( \epsilon^s_t \)) were not included in the information set used to determine expectations. Suppose now that agents observe \( \epsilon^d_t \) when they form their expectations (i.e. the current demand shock is part of the information set \( I_{t-1} \)). What will the reduced form for output look like in this case? Explain the economics.

3. The following problem is a bit more difficult than what you might expect on the mid-term, but it is good practice for understanding unstable roots and forward solutions. Consider Cagan’s model of the price level, which is familiar from class:

\[ m_t - p_t = \alpha(E_t p_{t+1} - p_t) \]

where \( m \) is the natural log of nominal money, \( p \) is the log of the price level, and \( E_t \) denotes the rational expectation operator conditional on information up to and including time \( t \). Assume that \( \alpha < 0 \), and defining \( \beta = \frac{\alpha}{1 - \beta} \), assume that \( \lim_{k \to \infty} \beta^k E_t p_{t+k} = 0 \). Finally, suppose that \( m \) is exogenous and is determined by the following process, where \( \gamma \) is positive but less than or equal to one and \( \epsilon \) is a zero-mean random variable:

\[ m_t = \gamma m_{t-1} + \epsilon_t. \]
a. Note that the second equation is a first-order difference equation in $m_t$. Keeping in mind that the expected value of $\epsilon$ is zero, prove that $E_t m_{t+i} = \gamma^i m_t$.

b. We know from class that Cagan’s model implies an unstable difference equation for the price level, and we provided a 'forward' solution to that difference equation. Use the result of part (a) in this solution to show how the equilibrium price level $p_t$ depends on $m_t$.

c. Now, using your knowledge of difference equations, go back to the money difference equation and solve for $m_t$ as a function of current and past $\epsilon_t$. Use this result in part (b). If you’ve done this correctly, you will have an expression for the equilibrium price at time $t$ as a function of current and past $\epsilon$s.

d. Discuss the implied dynamics of this model; in particular, what are the dynamic multipliers? Can you come up with an explanation of the economics underlying these multipliers?

e. How does your answer to (d) change if $\gamma = 1$?