A standard result in the two-sector dependent economy model is that demand shocks, such as government spending changes have no long-run impact on the real exchange rate. The intuition behind this result is that in these models, the real exchange rate is determined solely by supply-side factors. In this assignment, you will evaluate this result in the scenario when government spending plays a productive role in the economy by financing infrastructure spending which, in turn, is a factor of production complementary to both private capital and labor.

An infinitely-lived representative agent maximizes utility from the consumption of a traded and non-traded good:

$$\int_0^\infty \frac{(C^T C_N^\theta)^\gamma}{\gamma} e^{-\theta t} dt, \quad -\infty < \gamma < 1, 0 < \theta < 1$$

The agent accumulates wealth and physical capital, the accumulation rates for which are given by

$$\dot{b} = rb + Y_T - C_T + p[Y_N - C_N - I] - T_L$$

$$\dot{K} = I$$

Production takes place in the traded and non-traded sectors, according to

$$Y_T = A_T K_T^\alpha (L_T K_g^\eta)^{1-\alpha}, \quad 0 < \alpha < 1, 0 < \eta < 1, A_T > 0$$

$$Y_N = A_N K_N^\beta (L_N K_g^\eta)^{1-\beta}, \quad 0 < \beta < 1, 0 < \eta < 1, A_N > 0$$

The agent has an endowment of one unit of labor and $K$ units of private physical capital that he/she allocates to employment in both production sectors. $K_g$ represents the economy-wide stock of infrastructure capital provided by the government and accumulates according to

$$\dot{K}_g = g(Y_T + pY_N) - \delta_g K_g$$

In the above specification, $g$ represents the fraction of GDP (given by $Y_T + pY_N$) that is allocated to new public investment by the government and $\delta_g$ is the rate of depreciation of the infrastructure capital (for simplicity we have assumed that private capital does not depreciate). The government finances this expenditure by lump-sum tax revenues, while keeping its budget balanced:

$$g(Y_T + pY_N) = T_L$$

Derive the economy’s macroeconomic equilibrium and examine whether in this set-up, the real exchange rate ($p$) will be affected by changes in government spending on infrastructure, both in the long run as well as the short run.