Infrastructure Provision and Macroeconomic Performance

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Abstract

This paper studies the differences between economies where infrastructure is privately provided and those in which the government is the sole provider. Endogenous capital utilization decisions and their role in determining market prices for private capital and infrastructure provide a critical channel through which fiscal policy affects macroeconomic performance. The mode of infrastructure provision affects the degree to which capital utilization decisions can be internalized by the private sector, thereby impacting on an economy’s response to fiscal shocks. A targeted subsidy to private providers of infrastructure is unambiguously preferable to direct government provision (in terms of welfare), irrespective of how the subsidy or expenditure is financed or externalities such as congestion. In fact, the case for private provision is much stronger in economies characterized by high levels of congestion in infrastructure services. An income tax is more distortionary when both private capital and infrastructure are privately provided, than in the case where the government assumes responsibility for infrastructure provision. The choice between private and government provision also has a crucial effect on the design of optimal fiscal policy.

Keywords: Infrastructure Provision, Capital Utilization, User Cost, Fiscal Policy, Public Capital, Economic Growth

JEL Classification: D9, E2, E6, H2, O4

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1 Introduction

The recent policy shift in developing countries towards market provision of many goods and services traditionally viewed as "public goods" has once again put into question the role of government in economic progress. One such public good where the shift from government to private provision has been quite dramatic over the last two decades is infrastructure, which include roads, power, water and sewerage, irrigation, transportation and communications. While privately provided infrastructure services are quite common in the developed world, their provision in the developing world is still perceived to be in the government’s domain. This perception, however, is rapidly changing. The 1990’s witnessed the first major shift from government to private provision, with 132 low- and middle-income countries transferring about 2,500 infrastructure projects to the private sector. A recent World Bank study estimates that between 1990-2002, private sector commitments to infrastructure in developing countries totalled about $805 billion, or $62 billion per year, and accounted for about 25 percent of total infrastructure spending (Estache, 2004).\(^1\) The privatization of infrastructure in developing countries has mainly been driven by a rapid growth in demand, increasing public disenchantment with the performance and quality of state-provided services, and a policy shift towards fiscal restraint, especially after the debt and fiscal crises of the 1980s.\(^2\) Though infrastructure has been widely regarded by economists and policy-makers as an essential ingredient for growth and development, very little attention has been paid to the question of its provision in the growth literature.\(^3\)

This paper studies the differences between economies where infrastructure is privately provided and those in which the government is the sole provider, and how these differences, in turn, determine the impact of fiscal policy on macroeconomic performance and welfare. Specifically, we focus on the role played by endogenous utilization and depreciation of both infrastructure and private capital in determining the choice between private and government provision of infrastructure. This naturally raises the question: why is capital utilization important for the provision of infrastructure? The answer lies in the link between the pricing and utilization of infrastructure capital. Most privately-provided infrastructure services are excludable, implying that services may not be available to all users, possibly due to a pricing mechanism (e.g. user fees and tolls) and it is therefore important

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1 Canning and Bennathan (2000) and Briceno et al. (2004) estimate that in developing countries, the expected returns from private investment in telecommunications are between 30-40 percent, while the corresponding returns in electricity generation and road construction are 40 and 80 percent, respectively.

2 One excellent example can be found in India which, till the early 1990’s, was basically a closed economy with a huge public sector that operated on strong socialist principles. During the 1990’s phase of economic liberalization, infrastructure provision and privatization became contentious issues. While the telecommunications sector has been largely privatized, the nation’s major airports, power and electricity generation are also currently under privatization. The newly proposed interstate highway system will also be privately built and operated, with tolls and user fees being the main instruments of financing. A recent survey of India’s economic reforms can be found in Ahluwalia (2002).

3 The voluminous empirical literature on the productivity impact of infrastructure or "public capital" started with the findings of Aschauer (1989), and an early review can be found in Gramlich (1994). The formal treatment of public investment in theoretical models can be traced to Arrow and Kurz (1970) in the context of the neoclassical growth model and Barro (1990) for the endogenous growth model.
to understand the determinants of such a mechanism. We argue that the market price of infrastructure must be linked not only to its own usage, but to the usage of private capital. We formalize the concept of "usage" by introducing endogenous utilization decisions for both private capital and infrastructure, and linking them to the corresponding depreciation rates. On the other hand, when the government is the sole provider of infrastructure, the underlying services may be non-excludable in nature. In this case, the absence of a market-driven pricing mechanism for infrastructure might have a significantly different impact on the decision to utilize both private capital and infrastructure which, in turn, will differentially affect resource allocation relative to a privatized system.

To compare our analysis to the existing literature, we employ a theoretical framework where under government provision, infrastructure capital is non-excludable, while under private provision they are excludable. Therefore, under private provision, a user (in our case, the representative agent) can internalize the effects of utilization of both private and infrastructure capital on the production of output, and has the flexibility to adjust resource allocation along these margins in response to a fiscal shock. The utilization rates of the two types of capital are shown to be interdependent along the equilibrium path and jointly determine their respective market prices. By contrast, when the government provides the entire stock of (non-excludable) infrastructure, it is consequently treated by the private agent as exogenously given. The agent, therefore, does not internalize the effect of its allocation decisions on the accumulation, utilization, and depreciation of the government-provided infrastructure capital. This behavioral difference between the two regimes leads to critical differences in their responses to fiscal policy shocks, which eventually translate into substantial differences in welfare gains or losses. Therefore, the introduction of endogenous utilization and (consequently) depreciation turns out to be critically important for the choice between private and public provision. To our knowledge, the link between capital utilization, infrastructure provision, and macroeconomic performance has not been studied in the literature. This paper, therefore, provides a unique approach to study this important public policy issue.

The accumulation and usage of infrastructure also generates some form of rivalry, where the services derived by an individual may be affected by services derived by others. We capture the notion of rivalry by introducing a congestion externality that is associated with the accumulation and utilization of the two capital stocks. Specifically, given the stock of private capital, an increase in the stock of infrastructure mitigates the effects of congestion (e.g. given the number of cars, if the number of lanes in a highway is increased) and has a positive impact on aggregate productivity.

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5 The concept of capital utilization refers to the intensity or frequency with which capital equipment is operated, and is a popular construct in the business cycle literature; see Keynes (1936) for a very early discussion, and Lucas (1970), Calvo (1975), and Greenwood et al. (1988) for more recent contributions. In the context of intertemporal growth, its use is less prevalent, though some recent studies by Imbs (1999), Dalgaard (2003), and Chatterjee (2005) have demonstrated its importance for the dynamics of growth and convergence. To our knowledge, there is no known study of capital utilization in models of public investment and growth.
Conversely, given the stock of infrastructure, if the usage of private capital increases, then the effects of congestion are enhanced, with a dampening effect on productivity. This raises another important public policy question that this paper addresses: are economies with a high degree of congestion better off with private provision or government provision of infrastructure, with respect to economic welfare?

Our paper is related to a small literature that has studied the provision of infrastructure (or "public capital") in the context of growth. The first mention of the possibility of private provision can be found in Glomm and Ravikumar (1997), though they do not provide any formal treatment. Devarajan et al. (1998) examine the choice between private and public provision by evaluating the distortions created by the tax system in financing either direct government provision or subsidies to private providers. Chatterjee (2007) generalizes the Devarajan et al. framework to the open economy and shows that the choice between private and government provision depends crucially on the economy’s structural parameters such as the elasticity of substitution in production and the size of externalities, as well as borrowing constraints in international capital markets. We distinguish our approach from the previous literature by focusing on certain aspects of infrastructure provision absent from previous analyses. First, we focus on the role of endogenous capital utilization in determining the desirability between private and government provision of infrastructure. Second, we examine the consequences of rivalry, by introducing congestion generated by the usage of private and infrastructure capital. Third, we adopt a more flexible fiscal structure by introducing lump-sum taxes and debt financing. This aspect is absent in previous studies on this issue, which focus primarily on the distortionary income tax as the sole instrument for financing investment in infrastructure. By allowing for non-distortionary sources of financing infrastructure, we can decouple the effects of spending and revenue, making the comparison between the two regimes more transparent. This feature of the model is novel due to two reasons: first, the general consensus in the literature has been that the choice between public and private provision of infrastructure matters only when the government does not have access to non-distortionary sources of financing. The introduction of lump-sum tax and government debt enables us to do a robustness check of this consensus. Second, having a more flexible fiscal structure permits us to address another important issue absent from the previous literature: the design of optimal fiscal policy across the different regimes of infrastructure provision.

The contribution of this paper lies not only in its modeling, but also in the results, which are derived from a numerical analysis of the theoretical framework. This yields a number of interesting hypotheses:

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6 There is a voluminous literature in public and development economics on the provision of public goods; see Bergstrom et al. (1987), Shleifer (1998), Besley and Ghatak (2001), and Brett and Weymark (2008). This literature, based mainly on microeconomic models, abstracts from issues of growth and investment.

7 In earlier studies, such as Devarajan et al. (1998) and Chatterjee (2007), the absence of lump-sum taxes or other non-distortionary financing instruments in the government’s budget constraint generates time-inconsistency in implementing optimal fiscal policy.
(i) If the government wants to stimulate infrastructure investment, a targeted subsidy to private providers yields significantly higher welfare gains than if the government were to directly provide the additional investment without any private involvement. This result is robust to both the underlying financing instrument (lump-sum versus distortionary taxation) and variations in the congestion externality. Therefore, asymmetric tax distortions (as in Devarajan et al., 1998) or structural conditions (as in Chatterjee, 2007) are not essential in comparing the two modes of infrastructure provision. Rather, the inherent differences in the extent to which excludability (or the lack of it) permits the internalization of utilization decisions and market prices form the crux of our explanation. We view this as a significant and interesting departure from earlier analysis, which assumed that if the government has non-distortionary financing instruments at its disposal, the choice between public and private provision is irrelevant.

(ii) In comparing an infrastructure subsidy to private providers with complete government provision, we find that the welfare gains from subsidizing the private provision of infrastructure increasingly dominate those from government provision as the degree of congestion increases. This implies that the case for private provision should be much stronger in economies characterized by high levels of congestion.

(iii) An income tax increase is more distortionary in a privatized economy than under government provision, because an increase in the income tax rate in a privatized economy reduces the market return to both private and infrastructure capital and causes adjustments in both capital stocks as well as their utilization rates. By contrast, under government provision, the agent takes infrastructure as exogenously given and therefore has only one margin of adjustment, which is private capital and its utilization.

(iv) The choice between private and government provision has a crucial effect on the design of optimal fiscal policy. Under private provision, the government needs an income tax-infrastructure subsidy combination to attain the first-best optimum. The income tax rate targets not only the level of congestion, but also its source: the usage of infrastructure by the private sector. By contrast, under government provision, infrastructure is treated by the private sector as exogenously given and therefore its market price and utilization rate are not internalized. This requires a time-varying income tax rate to correct distortions both in transition as well as the steady state.

The rest of the paper is organized as follows. Sections 2-4 lay down the analytical framework for the private and government provision models. Section 5 characterizes the differences in optimal fiscal policy across the two regimes of infrastructure provision. Section 6 conducts a numerical analysis of the theoretical models, while Section 7 concludes.
2 Analytical Framework

We consider $N$ identical and infinitely lived representative agents, who maximize utility from consumption according to

$$U = \int_0^{\infty} \frac{C^\gamma}{\gamma} e^{-\beta t} dt, \quad -\infty < \gamma \leq 1 \tag{1}$$

Each agent also produces output ($Y$) using its individual stocks of private capital ($K$) and infrastructure capital ($K_g$). The accumulation of private capital, defined as an amalgam of physical and human capital (as in Rebelo, 1991), is undertaken by the agent, while infrastructure capital may be provided either "privately" (by the representative agent, in which case it would be excludable) or "publicly" (by the government, where it would be non-excludable). The production function can be described as follows

$$Y = A (u_k K)^\eta (u_g K_g^s)^{1-\eta}, \quad 0 < \eta < 1 \tag{2}$$

where $u_k$ and $u_g$ represent the rates of utilization of private capital and infrastructure, respectively. Specifically, we assume that we assume that the productive services derived by the agent from the two capital stocks are proportional to their "usage." We also assume that the accumulation and usage of private capital and infrastructure generate a congestion externality for the aggregate economy, which the individual agent cannot internalize. We specify a form of relative congestion where the services derived from infrastructure capital, $K_g^s$, are proportional to (i) the agent’s stock of infrastructure capital ($K_g$), and (ii) the usage of the agent’s stock of private capital relative to its economy-wide aggregate stock, $K$:

$$K_g^s = K_g \left(\frac{u_k K}{K}\right)^{1-\sigma}, \quad 0 \leq \sigma \leq 1 \tag{2a}$$

where $\sigma$ measures the degree of relative congestion.

The aggregate utilized stocks of private and infrastructure capital are defined as

$$\bar{K} = N(u_K K) \tag{2b}$$

$$\bar{K}_g = N(u_g K_g) \tag{2c}$$

The rates of accumulation of each type of capital are given by

$$\dot{K} = I - \delta_k (u_k) K \tag{3a}$$

$$\dot{K}_g = G - \delta_g (u_g) K_g \tag{3b}$$

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*We define the rate of utilization (or usage) of a given type of capital stock (private or infrastructure) as the speed or intensity with which it is operated (for example, "workweek," "hours per day," etc.), as in Taubman and Wilkinson (1970) and Calvo (1975).
where \( I \) and \( G \) measure the flow of new investment into the two capital goods, respectively, and \( \delta_k \) and \( \delta_g \) denote their corresponding depreciation rates. A critical point to note here is that the rates of depreciation of each capital good depend on their respective rates of utilization:

\[
\delta_i(u_i) = \frac{u_i^{\phi_i}}{\phi_i}, \quad \phi_i > 1, \quad i = k, g
\]  

(4)

The parameters \( \phi_i \) (\( i = k, g \)) in (4) measure the elasticity of depreciation with respect to utilization of the underlying stock of capital.\(^9\) Finally, we assume that the accumulation of both types of capital is costly and involves convex costs of adjustment.

\[
\Gamma(I, K) = I \left[ 1 + \frac{h_1}{2} \frac{I}{K} \right]
\]  

(5a)

\[
\Omega(G, K_g) = G \left[ 1 + \frac{h_2}{2} \frac{G}{K_g} \right]
\]  

(5b)

The budget constraints for the representative agent and the government will depend on how infrastructure is provided in an economy, i.e., whether it is provided by the (i) private representative agent, (ii) government, or (iii) a social planner (in this case both capital goods are provided by the planner). The sections below describe each regime of infrastructure provision.

3 Private Provision of Infrastructure

Under this regime, the representative agent provides both private and infrastructure capital and chooses their respective rates of utilization. Since infrastructure is privately provided, it is an excludable good with a market price, similar to private capital. However, the agents’ failure to internalize the congestion externality associated with the two types of capital provides an incentive for government intervention. Such an intervention can take place through a wide array of fiscal instruments, namely the income tax \( (\tau_y) \), a tax on interest income \( (\tau_b) \) generated by the issue of government bonds \( (b) \), a lump-sum tax \( (T) \), and a subsidy targeted for infrastructure investment \( (s) \). The private agent’s flow budget constraint is given by

\[
\dot{b} = (1 - \tau_b) rb + (1 - \tau_y) Y - C - \Gamma(I, K) - (1 - s) \Omega(G, K_g) - T
\]  

(6a)

\(^9\phi_i = u_i \delta'(u_i)/\delta(u_i), \ i = k, g. \) Under this specification, the marginal depreciation cost of utilization of a capital stock, \( \delta'(u_i) \), is variable. Note that as \( \phi_i \to \infty, \delta_i(u_i) \to 0 \) and \( u_i \to 1 \). The conventional assumption in the growth literature is that of a constant depreciation rate, so that \( \delta_i(u_i) = 0 \) and \( u_i = 1 \). Equation (4) represents the "depreciation-in-use" function, which is a standard specification in many Real Business Cycle models; see Burnside and Eichenbaum (1996). In other words, the depreciation rates are endogenously determined from equilibrium, given the optimal choices of the capital utilization rates.
The government finances any excess of expenditures over tax revenues by issuing debt in the form of infinitesimally short government bonds. The evolution of government debt is described by

$$
\dot{b} = (1 - \tau_b) rb + s \Omega (G, K_g) - \tau_y Y - T
$$

(6b)

Finally, the relevant production function for the representative agent is

$$
Y = A (u_k K)^{1-\sigma(1-\eta)} (u_g K_g)^{1-\eta} (\dot{K})^{(\eta-1)(1-\sigma)}
$$

(7)

Combining (6a) and (6b) yields the economy’s aggregate resource constraint

$$
Y = C + \Gamma (I, K) + \Omega (G, K_g)
$$

(8)

The representative agent maximizes (1), subject to (3a), (3b), (6a), and (7), while taking note of (4), (5a) and (5b). It is important to emphasize here that though the aggregate relationships (2b) and (2c) are not internalized by the agent in performing its optimization, they hold in equilibrium. Also, in deriving the equilibrium conditions, we have normalized $N = 1$, without loss of generality. The optimality conditions are

$$
C^\gamma - 1 = \lambda
$$

(9a)

$$
A(1 - \tau_y)(1 - \sigma(1 - \eta)) \left( \frac{u_g K_g}{K} \right)^{1-\eta} \frac{u_k^{\eta-1}}{q_k} = u_k^{\phi_k - 1}
$$

(9b)

$$
A(1 - \tau_g)(1 - \eta) \left( \frac{u_k K}{K_g} \right)^{\eta} \frac{u_g^{\eta-1}}{q_g} = u_g^{\phi_g - 1}
$$

(9c)

$$
i = \frac{I}{Y} = \left( \frac{q_k - 1}{h_1} \right) \left( \frac{K}{Y} \right)
$$

(9d)

$$
g = \frac{G}{Y} = \left( \frac{q_g - 1}{h_2} \right) \left( \frac{K_g}{Y} \right)
$$

(9e)

$$
\beta - \frac{\lambda}{\lambda} = (1 - \tau_b) r
$$

(9f)

$$
\frac{\dot{q}_k}{q_k} + \frac{A(1 - \tau_y)(1 - \sigma)(1 - \eta)}{q_k} u_k^{\eta}(K_g/K)^{1-\eta} \frac{u_k^{\eta-1}}{q_k} + \frac{(q_k - 1)^2}{2h_1 q_k} - \delta_k(u_k) = (1 - \tau_y) r
$$

(9g)

$$
\frac{\dot{q}_g}{q_g} + \frac{A(1 - \tau_y)(1 - \eta)u_k^{\eta}(K_g/K)^{-\eta}}{q_g} + \frac{(q_g + s - 1)^2}{2(1 - s)h_2 q_g} - \delta_g(u_g) = (1 - \tau_b) r
$$

(9h)

The above optimality conditions can be interpreted as follows. (9a) equates the marginal utility
from consumption to that of private wealth, measured by the shadow price \( \lambda \). Equations (9b) and (9c) represent the optimal decisions regarding the utilization of the two capital stocks, respectively. (9b) equates the after-tax marginal benefit from utilizing private capital, valued by its shadow price \( q_k \), to its marginal depreciation cost. Similarly, (9c) equates the after-tax marginal benefit and cost of utilizing infrastructure.\(^{10}\) Two things must be noted here. First, the factor utilization decisions do not internalize the congestion externality, \( \sigma \), and hence are sub-optimal. Second, (9c) shows that the usage of infrastructure depends, among other things, on its shadow price, \( q_g \). (9d) and (9e) describe the allocation of output to investment in the two capital goods, respectively. These allocations depend (i) positively on the respective shadow prices and (ii) inversely on the respective average products. As a result, the fraction of output allocated to either capital good is time-varying along the transition path to the steady-state equilibrium. Equations (9f)-(9h) represent no-arbitrage conditions for consumption, private capital and infrastructure respectively, thereby ensuring an interior equilibrium allocation. Additionally, equations (9g) and (9h) describe the evolution of the shadow (market) price of each capital good, which are crucial for clearing their respective markets.

The optimality conditions can also be used to derive the equilibrium growth rates for private capital, infrastructure, and consumption, respectively:

\[
\Psi_k = \frac{\dot{K}}{K} = \frac{q_k - 1}{h_1} - \delta_k(u_k) \tag{10a}
\]

\[
\Psi_g = \frac{\dot{K}_g}{K_g} = \frac{q_g + s - 1}{(1 - s)h_2} - \delta_g(u_g) \tag{10b}
\]

\[
\Psi_c = \frac{\dot{C}}{C} = \frac{(1 - \tau_b)r - \beta}{1 - \gamma} \tag{10c}
\]

Note that the growth rates of both private capital and infrastructure depend on their respective utilization rates. Moreover, (10b) clearly illustrates the dual role played by the subsidy in influencing the evolution of the privately provided stock of infrastructure: it increases the shadow price and lowers the cost of installation, thereby encouraging its accumulation.

### 3.1 Macroeconomic Equilibrium

The presence of both private capital and infrastructure implies that the equilibrium path will be characterized by transitional dynamics. Therefore, we will describe the macroeconomic equilibrium in terms of the shadow prices \( q_k \) and \( q_g \) and the following stationary variables: \( z = K_g/K \), the ratio of infrastructure to private capital, and \( c = C/K \), the consumption-private capital ratio. The first step in deriving the macroeconomic equilibrium is the determination of the utilization rates

\(^{10}\)The shadow prices \( q_k \) and \( q_g \) are measured in terms of the (unitary) price of government bonds. Consequently, the shadow value of wealth, \( \lambda \), is used as a numeraire.
for private capital and infrastructure. These can be obtained by solving the static equilibrium conditions (9b) and (9c):

\[
\begin{align*}
    u_k &\equiv u_k (q_k, q_g, z) = \left[ \frac{\Delta_1}{q_k^{\phi_g + \eta - 1} q_g^{-1}} \right]^{\frac{1}{\xi}} \\
    u_g &\equiv u_g (q_k, q_g, z) = \left[ \frac{\Delta_2}{q_k^{\phi_k - \eta} q_g^{\eta}} \right]^{\frac{1}{\xi}}
\end{align*}
\]

(11a) (11b)

where, \( \zeta = -[\eta \phi_g + (1 - \eta) \phi_k - \phi_k \phi_g] \), \( \Delta_1 = [A(1 - \tau_y)]^{\phi_g} [1 - \sigma (1 - \eta)]^{\phi_k + \eta - 1} (1 - \eta)^{1-\eta} \), and \( \Delta_2 = [A(1 - \tau_y)]^{\phi_k} [1 - \sigma (1 - \eta)]^{\eta} (1 - \eta)^{\phi_k - \eta} \).

The utilization rates for each capital stock depends on (i) the shadow prices \( q_k \) and \( q_g \), (ii) the infrastructure-private capital ratio, \( z \), and (iii) the structural, policy, and externality parameters of the model. Also note that (11a) and (11b) immediately determine the equilibrium depreciation rate of each capital good, as well as their evolution over time. To get some intuition on the behavior of the utilization rates, consider the following partial derivatives, under the mild restriction that \( \zeta > 0 \):

\[
\begin{align*}
    \frac{\partial u_k}{\partial z} &= \frac{(\phi_g - 1)(1 - \eta)}{\zeta} \left( \frac{u_k}{z} \right) > 0, \quad \frac{\partial u_g}{\partial z} = \frac{\eta(1 - \phi_k)}{\zeta} \left( \frac{u_g}{z} \right) < 0 \\
    \frac{\partial u_k}{\partial q_k} &= \frac{(1 - \phi_g - \eta)}{\zeta} \left( \frac{u_k}{q_k} \right) < 0, \quad \frac{\partial u_g}{\partial q_k} = -\frac{\eta}{\zeta} \left( \frac{u_g}{q_k} \right) < 0 \\
    \frac{\partial u_k}{\partial q_g} &= \frac{(\eta - 1)}{\zeta} \left( \frac{u_k}{q_g} \right) < 0, \quad \frac{\partial u_g}{\partial q_g} = \frac{(\eta - \phi_k)}{\zeta} \left( \frac{u_g}{q_g} \right) < 0
\end{align*}
\]

Intuitively, an increase in the proportion of infrastructure relative to private capital enhances the marginal product of private capital (being complementary inputs in production), thereby increasing its rate of utilization. On the other hand, in the presence of diminishing returns, an increase in infrastructure reduces its own average and marginal product, leading to a decline in its own rate of utilization. The respective utilization rates are negatively related to the shadow (market) prices of the two capital goods, indicating that an increase in market prices makes investment in either type of capital costly, thereby inducing the agent to reduce their rates of utilization, and thereby the cost of depreciation.

The core equilibrium dynamics can be expressed as

\[
\begin{align*}
    \dot{z} &= \frac{\dot{K}_g}{K_g} - \frac{\dot{K}}{K} = \left[ q_g + s - \frac{1}{(1 - s) h_2} - \delta_g(u_g) \right] - \left[ \frac{q_k - 1}{h_1} - \delta_k(u_k) \right] \\
    \dot{q}_k &= (1 - \tau_k) r q_k - A(1 - \tau_y) [1 - \sigma (1 - \eta)] u_k^{\eta} u_g^{1-\eta} z^{1-\eta} - \frac{(q_k - 1)^2}{2 h_1} + \delta_k(u_k) q_k
\end{align*}
\]

(12a) (12b)
\[ \dot{q}_g = (1 - \tau_b)rq_g - A(1 - \tau_y)(1 - \eta)u_k\eta u_q^{1 - \eta}z^{1 - \eta} - \frac{(q_g + s - 1)^2}{2(1 - s)h_2} + \delta_g(u_g)q_g \] 

(12c)

where \( u_k \) and \( u_g \) are given in (11). The evolution of the consumption-private capital ratio, however, is independent of the core dynamics:

\[ \frac{\dot{c}}{c} \equiv \frac{\dot{C}}{C} - \frac{\dot{K}}{K} = \left[ \frac{(1 - \tau_b)r - \beta}{1 - \gamma} \right] - \left[ \frac{\dot{q}_k - 1}{h_1} - \delta_k(u_k) \right] \] 

(12d)

The steady-state equilibrium is characterized by balanced growth and is attained when \( \dot{z} = \dot{q}_k = \dot{q}_g = \dot{c} = 0 \):

\[ \frac{\dot{q}_g + s - 1}{(1 - s)h_2} - \delta_g(u_g) = \frac{\dot{q}_k - 1}{h_1} - \delta_k(u_k) \] 

(13a)

\[ A(1 - \tau_y)[1 - \sigma(1 - \eta)]u_k^{\eta}u_q^{1 - \eta}z^{1 - \eta} + \frac{(\dot{q}_k - 1)^2}{2h_1\dot{q}_k} - \delta_k(u_k) = (1 - \tau_b)\tilde{r} \] 

(13b)

\[ A(1 - \tau_y)(1 - \eta)u_k^{\eta}u_g^{1 - \eta}z^{1 - \eta} + \frac{(\dot{q}_g + s - 1)^2}{2(1 - s)h_2\dot{q}_g} - \delta_g(u_g) = (1 - \tau_b)\tilde{r} \] 

(13c)

\[ \frac{(1 - \tau_b)\tilde{r} - \beta}{1 - \gamma} = \frac{\dot{q}_k - 1}{h_1} - \delta_k(u_k) \] 

(13d)

In addition, the aggregate resource constraint in the steady-state equilibrium is given by

\[ A\tilde{u}_k^{\eta}\tilde{u}_g^{1 - \eta}z^{1 - \eta} = \tilde{c} + \frac{\dot{q}_k^2 - 1}{2h_1} + \left[ \frac{\dot{q}_g^2 - (1 - s)^2}{2(1 - s)h_2} \right] \tilde{z} \] 

(13e)

Using (11a) and (11b) in (13a)-(13d), we can solve for the steady-state values of \( \tilde{z}, \tilde{r}, \dot{q}_k, \) and \( \dot{q}_g \). Substituting these values in the aggregate resource constraint (13e) immediately yields the steady-state level of the consumption-capital ratio, \( \tilde{c} \).\(^{11}\)

### 4 Government (Public) Provision of Infrastructure

We now consider a decentralized economy where the entire stock of infrastructure is provided by the government. Consequently, infrastructure capital is a public good whose services are non-excludable. The crucial difference between this regime and the private provision regime considered in section 3 is that the private agent takes the underlying stock of the government-provided infrastructure capital as exogenously given in making its allocation decisions. However, production is still subject to the congestion externality specified in section 2. The production

\(^{11}\)The linearized dynamics corresponding to this steady state can be described as \( \dot{X} = \Lambda \left( X - X' \right) \), where \( X' = (z, q_k, q_g), \tilde{X}' = (\tilde{z}, \tilde{q}_k, \tilde{q}_g), \) and \( \Lambda \) represents the 3x3 coefficient matrix of the linearized system. It can be numerically verified that the equilibrium is a saddle path with one stable (negative) and two unstable (positive) eigenvalues.
function under this regime is given by
\[ Y = A (u_K K)^{1-\sigma(1-\eta)} (\bar{u}_g K_g)^{1-\eta} (K)^{(\eta-1)(1-\sigma)} \]  
(14)

Note that, since the government directly provides infrastructure capital, we set \( \tilde{K}_g = \bar{u}_g K_g \) at the outset. Also, since the agent treats \( K_g \) as exogenously given, it does not internalize the utilization rate, \( u_g \), and its effect on the depreciation of infrastructure. As a result, the agent treats the depreciation of infrastructure capital as an \textit{exogenous} constant, \( \delta_g = \delta_g \), for all \( t \). The immediate consequence of this is that the marginal cost of infrastructure utilization is zero for the private agent. This implies that the rate of infrastructure utilization will also be treated by the private agent as an exogenous constant, i.e., \( u_g = \bar{u}_g \), for all \( t \) \( (0 < \bar{u}_g \leq 1) \). This is a key difference between this regime and the privatized one described in section 3.\(^{12}\)

The budget constraint for the private agent is given by
\[ \dot{b} = (1 - \tau_b) rb + (1 - \tau_y) Y - C - \Gamma (I, K) - T \]  
(15a)

Finally, the government’s budget constraint is
\[ \dot{b} = (1 - \tau_b) rb + \Omega (G, K_g) - \tau_y Y - T \]  
(15b)

Note that since the government provides infrastructure capital, its installation costs do not enter the private agent’s budget constraint (15a). The aggregate resource constraint continues to be given by (8).

The private agent maximizes (1), subject to (15a) and (3a), given (4a), (5a), and (14). As in section 3, we have normalized \( N = 1 \) and set \( \tilde{K} = u_k K \) (ex-post) in deriving the equilibrium conditions:
\[ C^{\gamma-1} = \lambda \]  
(16a)

\[ \beta - \frac{\dot{\lambda}}{\lambda} = (1 - \tau_b)r \]  
(16b)

\[ A(1 - \tau_y) [1 - \sigma(1 - \eta)] \left( \frac{\bar{u}_g K_g}{K} \right)^{1-\eta} \frac{u_k^{\eta-1}}{q_k} = \phi_k^{\phi_k-1} \]  
(16c)

\(^{12}\)A constant utilization rate for government-provided infrastructure can be easily rationalized. For example, consider electricity generation under government provision. In the absence of market prices that are required to equate supply and demand, a power plant may generate electric supply at a constant rate, irrespective of demand. Indeed, there is ample anecdotal evidence that most government-provided infrastructure services in developing countries are produced below capacity and have a disconnect with market demand.
\[ i = \frac{I}{K} = \frac{q_k - 1}{h_1} \quad (16d) \]

\[ \frac{\dot{q}_k}{q_k} + \frac{A(1 - \tau_y) [1 - \sigma(1 - \eta)] u_k^\eta (\bar{u}_g K_g/K)^{1-\eta}}{q_k} + \frac{(q_k - 1)^2}{2h_1 q_k} - \delta_k (u_k) = (1 - \tau_b) r \quad (16e) \]

The interpretation of the optimality conditions (16a)-(16e) is analogous to those in section 3. However, there are some key differences in the structure of the equilibrium. First, since the infrastructure utilization rate (\(\bar{u}_g\)) is exogenous to the private agent, a condition analogous to (9c) is now absent. Second, the evolution of the shadow price of infrastructure (\(q_g\)) is not a part of the macroeconomic equilibrium, since its accumulation is ensured by the government.

The evolution of the government-provided stock of infrastructure capital is given by (3b). To maintain an equilibrium of sustained growth, the government must spend a fixed fraction of aggregate output on infrastructure investment:

\[ \dot{K}_g = G - \delta g K_g, \quad G = gY, \quad 0 < g < 1 \quad (3b') \]

where \(g\) is government spending on infrastructure investment, as a fraction of aggregate output. Given the government’s budget constraint in (15b), this expenditure can be financed through a variety of policy instruments, such as income and lump-sum taxes, as well as government debt.

Another critical difference between the private and government provision models is that while \(g\) is an exogenous policy parameter under government provision, it is endogenous and time-varying under private provision; see (9e). Therefore, while both \(u_g\) and \(g\) are endogenously determined when infrastructure is privately provided, under government provision these become exogenous parameters. As we will see subsequently, this has important consequences for transitional dynamics, welfare and the design of optimal fiscal policy.

### 4.1 Macroeconomic Equilibrium

The basic structure of the macro-dynamic equilibrium remains similar to section 3. However, the equilibrium is now described in terms of \(z\), \(c\), and \(q_k\) only, and is independent of the shadow price of infrastructure, \(q_g\). The rate of utilization of private capital can be derived from (20c):

\[ u_k \equiv u_k(q_k, z) = \left[ \frac{A \{1 - \sigma(1 - \eta)\} (1 - \tau_y) (\bar{u}_g z)^{1-\eta}}{q_k} \right]^{1/\phi_{k-\eta}} \quad (17) \]

Comparing (17) with its counterpart (11a) under private provision, we see that when infrastructure is provided by the government, the choice of private capital utilization depends on (i) the shadow price of private capital, \(q_k\), and (ii) the ratio of infrastructure to private capital, \(z\), but is independent
of the shadow price of infrastructure as well as its utilization rate (since $u_g$ is constant in this regime).

The core equilibrium dynamics are given by

$$
\begin{align}
\ddot{z} &= \frac{K_g}{K_g} - \frac{\dot{K}}{K} = gA u_k i^{1-\eta}z^{1-\eta} - \gamma - \left[ \frac{q_k - 1}{h_1} - \delta_k(u_k) \right] \\
\dot{q}_k &= (1 - \tau_b)r - \frac{A(1 - \tau_y)\left[1 - \sigma(1 - \eta)\right]}{q_k} u_k^{\eta} i^{1-\eta}z^{1-\eta} - \frac{(q_k - 1)^2}{2h_1q_k} + \delta_k(u_k)
\end{align}
$$

where $u_k$ is given by (17). The evolution of the consumption-private capital ratio can be described as

$$
\begin{align}
\dot{c} &= \frac{\dot{C}}{C} - \frac{\dot{K}}{K} = \left[ \frac{(1 - \tau_b)r - \beta}{1 - \gamma} \right] - \left[ \frac{q_k - 1}{h_1} - \delta_k(u_k) \right]
\end{align}
$$

The economy will attain its balanced growth steady-state equilibrium when $\dot{z} = \dot{c} = \dot{q}_k = 0$. The corresponding steady-state conditions are

$$
\begin{align}
gA u_k i^{1-\eta}z^{1-\eta} - \gamma &= \frac{\dot{q}_k - 1}{h_1} - \delta_k(u_k) \\
\frac{(1 - \tau_b)r - \beta}{1 - \gamma} &= \frac{\dot{q}_k - 1}{h_1} - \delta_k(u_k) \\
\frac{A(1 - \tau_y)\left[1 - \sigma(1 - \eta)\right]}{q_k} u_k^{\eta} i^{1-\eta}z^{1-\eta} &= \frac{(\dot{q}_k - 1)^2}{2h_1q_k} - \delta_k(u_k) = (1 - \tau_b)r
\end{align}
$$

Equations (19a)-(19c) can be solved for $\dot{z}, \dot{q}_k$, and $\ddot{r}$. Given this solution, $\dot{c}$ can be determined from the economy’s aggregate resource constraint:13

$$
(1 - g)A u_k i^{1-\eta}z^{1-\eta} = c + \frac{q_k^2 - 1}{2h_1} + \frac{h_2}{2} \left[ g(A u_k i^{1-\eta}z^{1-\eta}) \right]^2
$$

5 A Centrally Planned Economy and Optimal Fiscal Policy

In a decentralized economy with externalities such as congestion, maximizing economic welfare might be an important objective for the government. This optimal policy intervention is attained by choosing the appropriate set of fiscal policy instruments that ensure an equilibrium allocation that replicates that of a centrally planned economy. The central planner’s allocation is always, by

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13The linearized dynamics corresponding to this steady state can be described as $\dot{J} = \Pi(\dot{J} - \dot{J}^*)$, where $\dot{J} = (z, q_k), \dot{J}^* = (\ddot{z}, \ddot{q}_k)$, and $\Pi$ represents the 2x2 coefficient matrix of the linearized system. The equilibrium is characterized by a saddle path with one stable and one unstable eigenvalue.
construction, the "first-best," since all externalities are internalized ex-ante. We therefore treat that allocation as a benchmark in deriving and comparing optimal fiscal policy rules for the private and government provision models.

The relevant production function for the central planner is given by

$$Y = A(u_k K)^\eta (u_g K_g)^{1-\eta}$$  \hspace{1cm} (20)$$

Note that in (20), the centrally planned economy is not subject to congestion, since the planner takes into account (2b) and (2c) \textit{ex-ante}. The central planner therefore maximizes (1), subject to the aggregate resource constraint (8) and the accumulation equations (3a) and (3b), given the production function in (20). Denoting all equilibrium variables in the centrally planned economy with a superscript "^\circ" (to denote their socially optimal levels), the optimal capital utilization rates are given by

$$\hat{u}_k = \left[ \frac{\Delta_1^* (\hat{z})^{(\phi_g-1)(1-\eta)}}{(\hat{q}_k)^{\phi_g+\eta-1}(\hat{q}_g)^{1-\eta}} \right]^{\frac{1}{\xi}}$$ \hspace{1cm} (21a)$$

$$\hat{u}_g = \left[ \frac{\Delta_2^* (\hat{z})^{\eta(1-\phi_k)}}{(\hat{q}_k)^{\phi_k}(\hat{q}_g)^{\phi_k-1}} \right]^{\frac{1}{\xi}}$$ \hspace{1cm} (21b)$$

where $\Delta_1^* = A^{\phi_g \eta \phi_g + \eta - 1} (1 - \eta)^{1-\eta}$, and $\Delta_2^* = A^{\phi_k \eta}(1 - \eta)^{\phi_k - \eta}$.

The corresponding steady-state conditions for a centrally planned economy are

$$\frac{\hat{q}_g - 1}{\hat{h}_2} - \delta_g(\hat{u}_g) = \frac{\hat{q}_k - 1}{\hat{h}_1} - \delta_k(\hat{u}_k)$$ \hspace{1cm} (22a)$$

$$\frac{(1 + \hat{v}\hat{g}) \eta A(\hat{u}_k)\eta (\hat{u}_g)^{1-\eta}}{\hat{q}_k} (\hat{z})^{1-\eta} + \frac{(\hat{q}_k - 1)^2}{2\hat{h}_1\hat{q}_k} - \delta_k(\hat{u}_k) = \hat{r}$$ \hspace{1cm} (22b)$$

$$\frac{(1 + \hat{v}\hat{g}) (1 - \eta) A(\hat{u}_k)\eta (\hat{u}_g)^{1-\eta}}{\hat{q}_g} (\hat{z})^{-\eta} + \frac{(\hat{q}_g - 1)^2}{2\hat{h}_2\hat{q}_g} - \delta_g(\hat{u}_g) = \hat{r}$$ \hspace{1cm} (22c)$$

$$\frac{\hat{r} - \beta}{1 - \gamma} = \frac{\hat{q}_k - 1}{\hat{h}_1} - \delta_k(\hat{u}_k)$$ \hspace{1cm} (22d)$$

In (22b) and (22c), $\hat{g}$ represents the fraction of output allocated to infrastructure investment by the central planner, and $\hat{v}$ denotes the corresponding shadow value (resource cost) of this allocation. In the case where the central planner chooses this fraction endogenously (optimally), $\hat{v} = 0$.

5.1 Optimal Fiscal Policy in the Private Provision Model

To derive optimal fiscal policy in the decentralized economy where infrastructure is privately provided, we compare the steady-state relationships (13a)-(13d), with the corresponding relationships
in the centrally planned economy, (22a)-(22d), while also comparing the capital utilization rates given in (11) and (21). We further assume that the central planner sets \( \hat{g} \) optimally, so that \( \hat{v} = 0 \) in (22b) and (22c). This enables us to solve for optimal fiscal policy in the private provision model. It is easy to verify that the optimal tax on bond income must be zero as there are no capital market imperfections, i.e., \( \hat{\tau}_b = 0 \). The optimal tax on capital income is given by

\[
\hat{\tau}_y = 1 - \left[ \frac{\eta}{1 - \sigma (1 - \eta)} \right]^{\phi_y - (1 - \eta)} \tag{23a}
\]

It is evident from (23a) that the optimal tax on income, \( \hat{\tau}_y \), must correct not only for congestion, \( \sigma \), but also for the effect of infrastructure usage on its depreciation, given by the elasticity parameter, \( \phi_y \). The intuition behind this result can be explained as follows. The presence of congestion raises the marginal product of private capital above its social rate of return. Therefore, a positive tax on capital income is required to equate the private and social returns. Additionally, the rate of infrastructure utilization, \( u_g \), is also affected by congestion; see (11b). Therefore, the congestion externality has a direct impact on the depreciation of infrastructure capital, through its effect on utilization decisions. The optimal tax on income, therefore, must take into account this dual effect of congestion. This feature of the optimal income tax rate distinguishes itself from existing results in the literature. To see this, consider the case where the rate of infrastructure depreciation is insensitive to its usage (a zero or constant depreciation rate). In our model, this special case can be approximated when \( \phi_y \to \infty \). It is easy to see from (23a) that

\[
\lim_{\phi_y \to \infty} \hat{\tau}_y = \frac{(1 - \sigma)(1 - \eta)}{1 - \sigma (1 - \eta)} \tag{23a'}
\]

The tax rate in (23a’) represents the standard result obtained in the theoretical literature on the relationship between congestion and taxation; see, for example, Turnovsky (1997). The specification in (23a) therefore is a more generalized expression for the optimal tax rate, which takes into account the effect of congestion on both the return to private capital and the depreciation of infrastructure. The conventional result found in the existing literature, such as (23a’), which does not take into account the effects of congestion on infrastructure depreciation, can then be viewed as a special case of our generalized model. It is easy to see from (23a) and (23a’) that when there is no congestion in the economy, i.e., \( \sigma = 1 \), the optimal income tax rate must be zero, i.e., \( \hat{\tau}_y = 0 \).

The optimal income tax rate described in (23a) is, however, insufficient to attain the first-best resource allocation, because the congestion externality affects the usage and accumulation of infrastructure capital as well, which in turn requires an additional corrective fiscal instrument. This is achieved through a targeted subsidy for infrastructure investment, given by

\[
\hat{s} = 1 - \frac{(\Sigma + 2q_g) - [\Sigma^2 + 4\Sigma q_g]}{2}^{1/2} \tag{23b}
\]
where,  
\[ \Sigma = 2h_2 \left\{ (\hat{r} - \beta)/(1 - \gamma) + \hat{\delta} \right\}^2 h_2 + A(\hat{\tau}_y - \eta)\hat{\delta}_y \hat{\delta}_g^{1 - \eta}/\gamma^\eta \].

The optimal infrastructure subsidy takes into account the equilibrium factor utilization rates and the shadow price of infrastructure, all of which are crucial determinants of its services. The optimal infrastructure subsidy serves two important objectives. First, it enables the private agent to internalize the social benefits from the aggregate stock of infrastructure in the presence of congestion. Second, it takes into account the effect of the accumulation of private capital and its usage on the usage and shadow price of infrastructure. Finally, we should note that the optimal tax and subsidy rates are constant over time. By substituting (23a) and (23b) into (12) and (13), it can be verified that an economy with private provision of infrastructure can not only replicate the steady-state equilibrium in a centrally planned economy, but also its dynamic adjustment path.

### 5.2 Optimal Fiscal Policy in the Public Provision Model

Under government provision of infrastructure, the burden of replicating the central planner’s allocation falls entirely on the tax system. Comparing (19a)-(19c) with (22a)-(22c), while noting the equilibrium choice of private capital utilization rates in (17) and (21), we can characterize optimal fiscal policy in the government provision model. As before, the optimal tax on bond income must be zero, i.e., \( \hat{\tau}_b = 0 \). This implies the following income tax rate in the steady-state equilibrium:

\[ \hat{\tau}_y = 1 - \frac{\eta}{1 - \sigma(1 - \eta)} (\hat{u}_k)^n (\hat{u}_g)^{1 - \eta} \]  

(24)

In deriving (24), we have assumed that the government in the decentralized economy sets \( g = \hat{g} \), i.e., the fraction of output allocated to infrastructure investment is set at its socially optimal level. When the government provides infrastructure capital, the optimal income tax rate must correct for three sources of externalities generated by the private sector: (i) the effects of congestion, (ii) the deviation of the private capital utilization rate from its socially optimal level, given by \( (\hat{u}_k)^n \), since the private agent does not internalize the contribution of infrastructure and its market price on its private capital utilization decision, and (iii) the deviation of the infrastructure utilization rate from its socially optimal level, given by \( (\hat{u}_g)^{1 - \eta} \) (note that we have normalized \( \hat{u}_g = 1 \)), since the private agent, by treating the stock of infrastructure as exogenous, fails to internalize the effect of its usage on infrastructure depreciation.

Comparing (23a) with (24), we see that the design of the optimal income tax policy depends critically on the underlying mode of infrastructure provision. For example, when there is no congestion, i.e., \( \sigma = 1 \), the optimal tax rate under private provision must be zero. However, under government provision, the tax rate must still be positive, since by not internalizing the utilization and productivity of infrastructure capital, the utilization rate for private capital deviates from its social optimum and hence requires a corrective policy intervention.

---

14 In deriving the optimal tax rate under government provision in (24), we have normalized the \( \hat{u}_g = 1 \), since it is a constant under government provision of infrastructure.
Another critical distinction between optimal fiscal policy under private and government provision relates to the dynamic properties of the optimal tax and subsidy rates. Under private provision, a constant income tax and infrastructure subsidy is sufficient to replicate both the transitional adjustment and the steady-state equilibrium of the central planner. However, under government provision, if $\tau_y$ is set as a constant according to (24), the adjustment path followed by the economy will fail to replicate that of the central planner. This happens because along the transition path in this regime, the private agent takes the stock of infrastructure as exogenously given and therefore does not internalize (i) the effect of its private investment decisions on the implied shadow (market) price of infrastructure and (ii) the utilization and depreciation rates of infrastructure capital and their consequences for private capital usage and accumulation. This behavioral aspect leads to an externality in private resource allocation along the transition to the steady-state equilibrium. A constant income tax rate cannot account for this transitional externality, and the economy converges to the first-best steady-state equilibrium at a non-optimal rate relative to the centrally planned economy. Therefore, $\tau_y$ in (24) is the first-best tax rate only in the steady state, not in transition.\textsuperscript{15}

This problem, however, can be corrected by a time-varying income tax rate, which takes the following form:

$$\tau_y(t) = \hat{\tau}_y + \theta \left[ z(t) - \tilde{z} \right]$$

(25)

The income tax rate in (25) tracks the dynamic evolution of the economy as the ratio of infrastructure to private capital changes in transition, thereby enabling the private agent to track the dynamic adjustment of the shadow price of infrastructure as well as its utilization rate.\textsuperscript{16}

In summary, we see that the mode of infrastructure provision (private or public) generates fundamental differences with respect to the design of optimal fiscal policy in a decentralized economy. These differences highlight the subtle, but crucial distinctions between the two regimes regarding the degree to which the various interdependencies between private and infrastructure capital are internalized by the private sector, both in transition and steady-state.

\textsuperscript{15}This transitional sub-optimality of the tax rate is typical in models where the dynamic adjustment path depends on the evolution of more than one state variable (in our case the two stocks of capital); see Benhabib et al. (2000) and Mino (2001).

\textsuperscript{16}The accurate determination of the constant $\theta$ is crucial for the first-best tax policy to replicate the dynamic adjustment of a centrally planned economy. To ensure this, the government must set $\theta$ such that $F(\mu, \theta) = V(\mu) = 0$, where $F(.)$ and $V(.)$ are polynomials derived from the determinants of the linearized matrix of coefficients in the centrally planned and decentralized economies, respectively, while $\mu$ is the stable eigenvalue in the centrally planned economy. When $\theta$ is chosen in this way, the speed of adjustment in the decentralized economy will replicate that of the centrally planned economy. Moreover, $\theta$ is only relevant along the transition path, and does not affect the steady-state equilibrium. As $z(t) \rightarrow \tilde{z}$, $\tau_y(t)$ will converge to its long-run optimal rate, $\hat{\tau}_y$. For a more elaborate proof, albeit in a different context, see Turnovsky (1997).
6 Private versus Government Provision: A Numerical Analysis

We begin our numerical analysis of the various models of infrastructure provision by specifying the benchmark structural and policy parameters, which have been chosen to be consistent with their corresponding empirical estimates.

<table>
<thead>
<tr>
<th>Benchmark Parameters</th>
</tr>
</thead>
<tbody>
<tr>
<td>Preference Parameters: $\gamma = -1.5, \beta = 0.04$</td>
</tr>
<tr>
<td>Production Parameters: $A = 1.5, \eta = 0.8, h_1 = h_2 = 15, \phi_k = \phi_g = 2$</td>
</tr>
<tr>
<td>Externality Parameter: $\sigma \in [0, 1]$</td>
</tr>
<tr>
<td>Policy Parameters: $\tau_y = 0, \tau_b = 0, s = 0$</td>
</tr>
</tbody>
</table>

As shown in the table above, the preference parameters $\beta$ and $\gamma$ yield an intertemporal elasticity of substitution in consumption equal to 0.4, consistent with recent empirical evidence; see Guvenen (2006). The output elasticity of private capital $(\eta)$ is set at 0.8, which is reasonable when we define private capital as an amalgam of physical and human capital, as in Romer (1986). This also implies that the corresponding elasticity for infrastructure is 0.2, which is within the empirically estimated range of 0.1 – 0.3; see Gramlich (1994). The adjustment cost parameters are consistent with Ortiguela and Santos (1997) and their equality serves as a plausible benchmark. While $A$ represents a scale parameter in the production function, the choice of $\phi_k$, the elasticity of depreciation with respect to private capital utilization is set at 2, following Basu and Kimball (1997). Since there is no known estimate for $\phi_g$, the corresponding elasticity with respect to infrastructure capital, we set it equal to $\phi_k$. The congestion parameter $(\sigma)$ is varied from 0 (proportional congestion) to 1 (no congestion). We set the tax and subsidy rates to zero in the laissez-faire economy, while under government provision we assume that the necessary public expenditure on infrastructure is financed by appropriately adjusting lump-sum taxes or government debt.

6.1 Benchmark Equilibrium

To compare the two regimes of infrastructure provision, we must start from a common benchmark equilibrium. To achieve this outcome, we note that in the government provision regime, (i) spending on infrastructure $(g)$, represents an arbitrary policy choice, and (ii) the utilization and depreciation rates for infrastructure capital $(\bar{u}_g$ and $\delta_g)$ are exogenous to private decisions. Therefore, we calibrate these exogenous variables in the government provision economy to equal their corresponding equilibrium values in the private provision (laissez-faire) regime. This strategy...

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17 There have been a few attempts in the literature to measure the elasticity parameter $\phi$, and the ones available show significant variation. For example, Burnside and Eichenbaum (1996) estimate $\phi_k = 1.56$ for U.S. manufacturing, while Finn’s (1995) estimate is 1.44. More recently, Dalgaard (2003) calculates $\phi_k$ to be about 1.7 for Denmark. Basu and Kimball (1997) note that the upper bound of the 95 percent confidence interval for $\phi_k$ is 2.
yields an identical long-run equilibrium allocation in the two regimes and provides us with a convenient starting point for analyzing the relationship between the mode of infrastructure provision, fiscal policy, and macroeconomic performance.

Table 1 depicts the benchmark equilibrium in the laissez-faire (private provision) economy which, as described above is also identical to the equilibrium under government provision. The benchmark equilibrium is characterized for three levels of congestion: $\sigma = 0$, 0.5, and 1. By construction, the case where there is no congestion, i.e., $\sigma = 1$, represents the first-best or social optimum for the economy. At the social optimum, the equilibrium ratio of infrastructure to private capital ($\tau$) is 0.25, while the shadow prices of infrastructure and private capital are equalized at 2.42. The equilibrium utilization and depreciation rates for private capital and infrastructure in the laissez-faire economy are also equal at about 38 and 7 percent, respectively. The consumption-output ratio is 0.53, while the allocation of output to private investment and infrastructure is about 22 and 5.5 percent, respectively. These equilibrium allocations imply a long-run growth rate of 2.34 percent. Since we have set our benchmark tax and subsidy rates to zero, our results should be viewed as a numerical illustration of the analytical framework developed in sections 2-4, rather than a calibration for a particular economy.

Table 1 also clearly illustrates how the equilibrium resource allocation in a laissez-faire economy is affected by congestion. As congestion increases ($\sigma \to 0$), the marginal return from private capital rises and that from infrastructure falls, relative to the social optimum. This leads to an over-investment in private capital and an under-investment in infrastructure by the private agent. Further, as congestion increases, the utilization of both capital stocks increase relative to their social optimum, which consequently raises their depreciation rates as well. The higher aggregate demand for investment reduces the consumption-output ratio and raises the long-run growth rate relative to their corresponding first-best rates.

### 6.2 Fiscal Policy and Transitional Dynamics

This section considers the effects of fiscal policies on the macro-dynamic equilibrium in the two regimes of infrastructure provision. In particular, we focus on the two fiscal instruments that have a direct impact on the incentives to invest: the infrastructure subsidy ($s$) and the income tax rate ($\tau_y$). We compare the effects of these instruments numerically by considering three policy experiments: (i) a targeted infrastructure subsidy under private provision versus an equivalent increase in public investment under government provision, both financed by lump-sum taxation, (ii) experiment (i), but when spending is financed by a distortionary income tax, and (iii) an increase in the income tax rate (with infrastructure investment constant). The corresponding results are

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18. Note that under government provision, the shadow price of infrastructure, $q_g$, does not apply, since the private agent takes the government-provided stock as exogenously given.

19. Most of the equilibrium values lie within their corresponding ranges for the OECD countries, as reviewed by Morshed and Turnovsky (2004).

20. Since the optimal tax on interest income is zero (from 17b), we set $\hat{\tau}_b = 0$ throughout our calibration exercises.
reported in Table 3 and Figure 1.\textsuperscript{21} As we will see, though the two regimes start with identical equilibrium allocations, fiscal policy shocks lead to distinct differences in both their transitional and long-run responses, which ultimately generate substantial differences in long-run welfare levels.

### 6.2.1 Infrastructure Subsidy versus Government Investment

Tables 2A and 2B compare the long-run response to an increase in the infrastructure subsidy in the private provision regime with a corresponding increase in public spending on infrastructure in the government provision regime. Specifically, we consider a permanent increase in the infrastructure subsidy rate from zero (the benchmark laissez-faire rate) to 10 percent of GDP. To compare the responses across the two regimes, we calibrate the corresponding increase in public investment under government provision to equal the increase in infrastructure investment under private provision (generated by the subsidy). The main difference between Tables 2A and 2B lies in the mode of financing the underlying investment in infrastructure in the two regimes: in Table 2A, the spending is financed by a lump-sum tax increase, while in Table 2B the financing instrument is the distortionary income tax, which is adjusted appropriately in accordance with the government’s budget constraint.

Under lump-sum tax financing (Table 2A), an infrastructure subsidy to private providers drives a wedge between the returns to the two capital stocks by lowering the cost of infrastructure investment and raising its market (shadow) price ($q_g$) relative to that of private capital ($q_k$).\textsuperscript{22} This generates a long-run increase in the allocation of output to infrastructure relative to private capital, reflected by an increase in $G/Y$ and a decline in $I/Y$. The higher return to infrastructure also increases its rate of utilization. Consequently, this raises the productivity of private capital and its rate of utilization as well. The increase in the rates of accumulation and utilization of the two capital stocks raises the equilibrium growth rate and increases the flow of output in a proportion larger than consumption (reflected by a decline in the consumption-output ratio).

Under government provision, however, an equivalent increase in public spending ($g$) raises the total cost of infrastructure investment, since the accumulation of infrastructure is subject to installation costs that must also be financed by the government. As a result, the financing burden for the government is larger than in the privatized economy, and there is a larger crowding out of private investment and consumption relative to the privatized economy, as the government uses the economy’s resources in accumulating and installing infrastructure. The fall in the consumption-output ratio is very large relative to that in the privatized economy. On the other hand, the large amount of resources devoted by the government to infrastructure (new investment and installation costs) raises the growth rate more than that under private provision.

\textsuperscript{21}Since both economies have a common pre-shock equilibrium, we report the long-run changes in each variable following an underlying fiscal shock. Therefore, if $x$ is an endogenous variable, we report $dx = x_1 - x_0$, where $x_1$ is the after-shock steady-state equilibrium value of $x$ and $x_0$ is its pre-shock level. The changes in the growth rates are reported as percentages.

\textsuperscript{22}Note here that since the infrastructure subsidy is tied to the cost of public investment (and not output), we calibrate it such that it equals 10 percent of GDP.
When the mode of financing is changed to the distortionary income tax (Table 2B), the above mechanisms remain qualitatively the same. However, since the income tax used to finance the subsidy under private provision lowers the after-tax marginal return on both types of capital, the crowding out of private investment is larger and the increase in infrastructure investment smaller than under lump-sum tax financing. The lower after-tax marginal return on the two types of capital causes a large substitution in favor of consumption, which leads to an increase in the consumption-output ratio under private provision. Since the higher income tax is being used to finance infrastructure investment, it causes modest increases in the rates of utilization and long-run growth. Under government provision, since infrastructure is publicly provided, the extent of crowding out of private investment is smaller, thereby leading to a more expansionary effect on the growth rate than under private provision.

**Transitional Dynamics**

The dynamic adjustment in the two regimes in response to an increase in the subsidy rate and in government investment is illustrated in figure 1 (panels A and B, respectively). We illustrate the case of lump-sum tax-financing only, since income tax-financing has qualitatively similar implications.

The transitional dynamics across the two regimes are dramatically different, although their long-run responses are qualitatively similar. In the private provision model, an infrastructure subsidy generates a substantial increase in expected long-run productivity. However, since the stock of infrastructure cannot be changed instantaneously, the private agent responds by increasing the rate of its utilization, \( u_g \). As a result, \( u_g \) jumps up instantaneously to overshoot its higher long-run equilibrium (fig. 1Ai). The higher expected long-run stock of infrastructure also raises the expected long-run productivity of private capital, causing its rate of utilization, \( u_k \), to jump up as well, but by less than the jump in \( u_g \), to maintain equality in their respective rates of return. Thereafter, as infrastructure is accumulated, its average product falls and \( u_g \) gradually declines and approaches its new steady-state equilibrium rate from above. At the same time, the rising stock of infrastructure raises the productivity of private capital, and \( u_k \) increases in transition to approach its new (common) equilibrium rate from below. In the new steady-state equilibrium, both capital stocks have the same rates of utilization. By contrast, in the government provision model, the private agent cannot alter the utilization rate of infrastructure, since it is exogenous to private decisions. As a result, \( u_g \) in the government provision model remains unchanged throughout transition. The higher government spending therefore leads to a slight downward jump in the utilization rate for private capital (fig. 1Bi). Thereafter, as infrastructure is accumulated, the positive productivity effect on private capital raises its utilization rate gradually to its new, higher steady-state equilibrium.

Figures 1A and B (ii) illustrate the dynamic response of the private investment-output ratio in the two regimes. In the private provision model, the increase in infrastructure investment due to the subsidy requires a substitution away from private capital investment, leading to an
instantaneous decline in $I/Y$. The increase in infrastructure investment causes output to grow at a much faster rate than private capital in transition, causing the private investment-output ratio to gradually decline to its lower after-shock equilibrium level. In the government provision model, the appropriation of private resources for infrastructure investment by the government (due to installation costs) causes the agent to increase its allocation to private investment to maintain the flow of output. This causes an initial upward jump in $I/Y$. Thereafter, as public capital accumulates, output grows much faster than private capital, and the investment-output ratio falls sharply to its lower steady-state equilibrium.

The response of the consumption-output ratio is depicted in figures 1A and B (iii). Again, we see that the dynamic responses across the two regimes are dramatically different. In the private provision economy, the higher long-run productivity of private capital (due to infrastructure accumulation and higher utilization) causes the consumption-output ratio to increase instantaneously. Thereafter, the investment boom causes output to increase at a rate higher than consumption, so that the consumption-output ratio falls in transition. In the government provision model, consumption is crowded out instantaneously as the agent tries to offset the higher government spending by substituting away from consumption into private investment. This is shown by the large downward jump of the consumption-output ratio. Thereafter, the growth in output and private capital (due to infrastructure investment by the government) causes this ratio to increase over time, indicating that the benefits of investment and growth are reallocated somewhat back to consumption during transition to make up for the large initial decline.

6.2.2 An Income Tax Increase

Table 2C reports the long-run effects of a 10 percent increase in the income tax rate across the two regimes, from its benchmark rate of $\tau_y = 0$ to 0.1. In general, an increase in the tax on income will reduce the after-tax returns on both private capital and infrastructure in the privatized economy. This leads to a substitution away from investment in both types of capital towards consumption. As a result, the consumption-output ratio increases, while the allocations to the two types of investment fall. The allocation to private investment declines more relative to infrastructure because, given their respective output elasticities, private investment has a larger initial share in output. On the other hand, the lower after-tax marginal product of the two capital stocks causes the private agent to lower the respective utilization rates ($u_k$ and $u_g$) which, by reducing depreciation costs, partially offsets the decline in investment. Overall, the decumulation of the two capital stocks and their lower utilization lead to a decline in the long-run equilibrium growth rate.

Under government provision, the private agent cannot alter investment in infrastructure or its utilization. Since these variables are exogenous to the private agent, the burden of adjustment falls entirely on the stock of private capital. As a result, the decline in the private investment-output ratio is larger than in the privatized economy. However, since the government maintains its rate of investment in infrastructure (unlike the privatized economy, where it declines), the decline in
productivity of private capital is less than that under private provision (due to complementarity between the two types of capital). Consequently, the reduction in the rate of utilization of private capital is also less than that in the privatized economy. This implies a smaller substitution towards consumption, and consequently, a smaller decline in the equilibrium growth rate relative to the privatized economy.

**Transitional Dynamics (or the Lack of It)**

In contrast to the effects of tax shocks in models with fixed depreciation and capital utilization rates (where typically $u_k = u_g = 1$), there is little or no dynamic adjustment when these variables are endogenous. For example, in the private provision regime, the income tax shock does not generate any dynamic adjustment and the economy immediately jumps to its after-shock steady-state equilibrium. This is a surprising result, since traditionally models with multiple capital stocks display slow adjustment to tax shocks; see Futagami et al. (1993). However, when one considers the role of capital utilization and depreciation in resource allocation, this result is not difficult to rationalize. Since the utilization decisions provide the agent an extra margin along which the flow of output can be maintained, the ratio of public to private capital, $z$, remains invariant to a tax increase. This happens because, on the margin, the tax shock affects the returns to either type of capital symmetrically. Also, the no-arbitrage conditions (9g) and (9h) imply that the after-tax equality in the return to both types of capital must be maintained at all points of time. This is ensured as the agent, in response to the tax increase, reallocates resources away from investment in the two capital goods, but readjusts their respective utilization rates to maintain $z$ at a constant level. Therefore, although the equilibrium allocations and the growth rate change, the effect of the tax increase on the economy is instantaneous. In the government provision model, since one adjustment margin is not available to the agent (infrastructure investment and its utilization), a tax shock does generate a dynamic response, but the adjustment is very quick as the agent appropriately adjusts the utilization rate of private capital. Therefore, we have chosen only to discuss the qualitative nature of the dynamics, rather than illustrate them graphically.

**6.2.3 Welfare Effects of Fiscal Shocks and the Role of Capital Utilization**

One crucial norm for comparing the relative performance of the two regimes of infrastructure provision is the response of economic welfare to the fiscal policy changes considered in the previous sections. It is also instructive to check the robustness of the welfare responses to variations in the congestion parameter. The results of these exercises are reported in Table 3.

The first thing to note about Table 3 is that a targeted subsidy to private providers of infrastructure yields uniformly higher welfare gains than an equivalent increase in government investment, irrespective of whether the financing instrument is a lump-sum tax or an income tax. This comparison is also robust to changes in the congestion externality. This is a significant result, since it has been generally thought that if the government has access to non-distortionary sources of financing such as lump-sum taxes, the choice between private and public provision of infrastructure
is irrelevant (in terms of welfare) as the government can do at least as well as the private sector; see Devarajan et al. (1998). However, our results indicate that even with lump-sum taxes and debt-financing at its disposal, the government cannot outperform the private sector in the context of infrastructure provision. While much of the earlier analysis of this issue has relied heavily on the asymmetric distortions created by the income tax in financing infrastructure investment in the two regimes, our results indicate that the underlying mode of financing or the government’s access to non-distortionary sources of financing does not matter for the choice between private and public provision of infrastructure. Instead, one must look beyond tax distortions to explain the inherent differences between the two modes of infrastructure provision. As we discuss below, the explanation lies in the behavioral differences between the two regimes, characterized by the asymmetric response of capital utilization decisions to the underlying fiscal policy shocks.

Under private provision, the subsidy lowers the cost of infrastructure investment and increases the rates of utilization of both the capital stocks. The resultant flow of output is therefore much higher than under government provision, where the private agent cannot alter the rate of infrastructure utilization to its advantage. Moreover, an increase in direct government spending on infrastructure increases installation costs, which leads to a larger crowding out of private consumption. On the other hand, under private provision, the ability to control the allocation of resources to both the capital stocks by adjusting their rates of utilization requires a much smaller adjustment in consumption. These behavioral responses lead to dramatic differences in welfare levels across the two regimes. For example, in the absence of congestion ($\sigma = 1$), the welfare gain from a lump-sum tax-financed subsidy to private providers is 29.57 percent, while that from an equivalent increase in government investment is only 1.52 percent. Further, an increase in congestion leads to increasing welfare losses under government provision, while there are increasing gains under private provision. This happens because, when congestion is higher, the substitution away from consumption under government provision is also higher, as the agent accumulates more private capital in order to increase the services from the stock of infrastructure. Therefore, infrastructure accumulation under government provision makes the effects of congestion worse, reducing welfare in the long run. By contrast, a subsidy under private provision causes a smaller substitution away from consumption into private investment, as the agent can adjust both the capital utilization rates to derive the necessary services from infrastructure. This lowers the adverse effects of congestion and makes the economy better off. This comparison between private and government provision is qualitatively robust to the case where the increase in infrastructure spending (private or public) is financed by a distortionary income tax (Table 3, Row 2). Therefore, it is evident from the first two rows of Table 3 that the mode of financing infrastructure investment (lumpsum versus income tax) in the two regimes is not sufficient to understand the differences in welfare changes. Instead, the differences in the response of utilization decisions across the two regimes is critical for understanding the equilibrium welfare responses in the two regimes.

When the two economies are subject to an income tax shock (Table 3, Row 3), the comparisons
between the two regimes are now reversed: the economy under government provision yields uniformly higher welfare gains or lower welfare losses relative to the privatized economy. This happens because a tax shock is more contractionary for the privatized economy. By reducing the after-tax return on both private capital and infrastructure, a tax shock leads not only to lower aggregate investment in the privatized economy, but also to lower utilization rates for both capital stocks. As a result, the flow of output declines, yielding only modest welfare changes. In comparison, a tax increase under government provision reduces only the return and utilization of private capital. Moreover, since the government maintains its infrastructure investment rate, the resultant fall in output is partially offset. This leads to higher welfare gains or lower losses than under private provision. For example, when $\sigma = 1$, a 10 percent increase in the income tax rate lowers welfare by 0.88 percent in the privatized economy, while the corresponding loss under government provision is lower at 0.53 percent. Therefore, an income tax is more distortionary under private provision than under government provision. As the level of congestion increases, a tax increase under government provision has a larger positive impact than under private provision, since the government, by maintaining its expenditure on infrastructure, ensures that the corresponding services derived by the private agent are unaffected. Interestingly, this result contrasts with that of Devarajan et al. (1998), where tax distortions are higher under government provision. Endogenous utilization decisions, therefore, serve as the crucial link between our results and those in their paper.

To summarize, the desirability of a particular mode of infrastructure provision depends not on the magnitude of externalities (congestion) or distortions created by the tax system (lump-sum versus income tax), but rather on the differences in the private sector’s ability to internalize capital utilization rates and market prices in response to fiscal policy shocks when infrastructure is privately or publicly provided.

7 Conclusions

The choice between private versus public provision of infrastructure has recently become a contentious issue in policy circles. However, it has received very little formal attention from economists. This paper builds on a very small but promising literature and attempts to provide some insights into the choice between private and government provision and its impact on an economy’s aggregate performance. Specifically, we focus on endogenous capital utilization decisions and their role in determining market prices for private capital and infrastructure as a crucial channel through which the choice between private and public provision can be better understood.

The contributions of this paper lie both in its approach and results. The link between capital utilization, infrastructure provision, and macroeconomic performance has not been studied previously in the growth literature. Our analysis, therefore, provides a unique approach to study this important public policy issue. Specifically, by introducing both private and infrastructure capital utilization as decision variables, we have illustrated how the mode of infrastructure provision
affects the degree to which these decisions are internalized and how, in turn, they affect an economy’s response to fiscal shocks. We show that the choice between private and public provision of infrastructure matters even when the government has access to lump-sum taxes or other non-distortionary fiscal instruments. Indeed, the central result of the paper is that from the standpoint of economic welfare, a targeted or tied subsidy to private providers of infrastructure is unambiguously preferable to direct government provision, irrespective of how the subsidy or government expenditure is financed. This result is also robust to variations in the congestion externality. The robustness of our results to financing instruments and the magnitude of the externality represents a significant departure from previous work in this area.

Another contribution of this paper is to show that the mode of infrastructure provision has an important bearing on the design of optimal fiscal policy in a growing economy. While under private provision, both an (constant) income tax and an infrastructure subsidy are jointly required to attain the first-best equilibrium, under government provision the burden of attaining optimality falls on the income tax alone, which in turn needs to be time-varying.

Finally, our numerical experiments yield a few other interesting hypotheses, which can be easily taken to the data. For example, the welfare gains from subsidizing private provision of infrastructure increase relative to direct government provision as the degree of congestion increases. This implies that for economies with high degrees of congestion, private provision might be a better alternative to public provision of infrastructure. Further, we show that an income tax is more distortionary when both private capital and infrastructure are privately provided, than in the case where the government assumes responsibility for infrastructure provision. Again, these results stem from the effect a particular mode of infrastructure provision has on capital utilization decisions and depreciation rates.

In future research, the link between infrastructure provision and macroeconomic performance can be explored in several promising directions. For example, one can examine the effect of the choice between private and public provision on several issues of concern to policy-makers in both developed and developing countries, namely labor productivity, possibilities of joint public-private partnerships, and the distribution of income.
TABLE 1
Benchmark Equilibrium in the Laissez-faire Economy for Different Levels of Congestion

\[ A = 1.5, \gamma = -1.5, \beta = 0.04, h_1 = h_2 = 15, \phi_k = \phi_g = 2 \]
\[ \tau_y = \tau_b = s = 0 \]

<table>
<thead>
<tr>
<th>( \sigma )</th>
<th>( \bar{z} )</th>
<th>( \bar{q}_j )</th>
<th>( \bar{u}_j )</th>
<th>( \delta_j )</th>
<th>( C/Y )</th>
<th>( I/Y )</th>
<th>( G/Y )</th>
<th>( \Psi ) (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \sigma = 0 )</td>
<td>0.20</td>
<td>2.69</td>
<td>0.404</td>
<td>0.082</td>
<td>0.433</td>
<td>0.256</td>
<td>0.0512</td>
<td>3.08</td>
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<tr>
<td>( \sigma = 0.5 )</td>
<td>0.22</td>
<td>2.56</td>
<td>0.391</td>
<td>0.077</td>
<td>0.481</td>
<td>0.239</td>
<td>0.0531</td>
<td>2.72</td>
</tr>
<tr>
<td>( \sigma = 1 )</td>
<td><strong>0.25</strong></td>
<td><strong>2.42</strong></td>
<td><strong>0.376</strong></td>
<td><strong>0.071</strong></td>
<td><strong>0.530</strong></td>
<td><strong>0.220</strong></td>
<td><strong>0.055</strong></td>
<td><strong>2.34</strong></td>
</tr>
</tbody>
</table>

Note: \( j = k, g \)
\( \bar{\Psi} \) = Long-run (balanced) growth rate
TABLE 2

Private versus Government Provision of Infrastructure: Steady-State Effects of Fiscal Shocks

\( (\sigma = 1) \)

A. Subsidy to Private Providers versus Government Spending (Lump-sum Tax-Financed )

<table>
<thead>
<tr>
<th></th>
<th>( d(C/Y) )</th>
<th>( d(I/Y) )</th>
<th>( d(G/Y) )</th>
<th>( d\bar{u}_k )</th>
<th>( d\bar{u}_g )</th>
<th>( d\Psi )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Private Provision Model</td>
<td>-0.18</td>
<td>-1.20</td>
<td>+4.91</td>
<td>+2.14</td>
<td>+2.14</td>
<td>+0.57</td>
</tr>
<tr>
<td>Government Provision Model</td>
<td>-8.99</td>
<td>-1.24</td>
<td>+4.91</td>
<td>-2.23</td>
<td>-</td>
<td>+0.59</td>
</tr>
</tbody>
</table>

B. Subsidy to Private Providers versus Government Spending (Income Tax-Financed )

<table>
<thead>
<tr>
<th></th>
<th>( d(C/Y) )</th>
<th>( d(I/Y) )</th>
<th>( d(G/Y) )</th>
<th>( d\bar{u}_k )</th>
<th>( d\bar{u}_g )</th>
<th>( d\Psi )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Private Provision Model</td>
<td>+3.93</td>
<td>-2.26</td>
<td>+4.38</td>
<td>+0.76</td>
<td>+0.76</td>
<td>+0.20</td>
</tr>
<tr>
<td>Government Provision Model</td>
<td>-6.14</td>
<td>-1.67</td>
<td>+4.38</td>
<td>+1.36</td>
<td>-</td>
<td>+0.35</td>
</tr>
</tbody>
</table>

C. Income Tax Increase

<table>
<thead>
<tr>
<th></th>
<th>( d(C/Y) )</th>
<th>( d(I/Y) )</th>
<th>( d(G/Y) )</th>
<th>( d\bar{u}_k )</th>
<th>( d\bar{u}_g )</th>
<th>( d\Psi )</th>
</tr>
</thead>
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<tr>
<td>Private Provision Model</td>
<td>+4.90</td>
<td>-1.41</td>
<td>-0.35</td>
<td>-1.58</td>
<td>-1.58</td>
<td>-0.39</td>
</tr>
<tr>
<td>Government Provision Model</td>
<td>+4.05</td>
<td>-1.48</td>
<td>-</td>
<td>-1.44</td>
<td>-</td>
<td>-0.36</td>
</tr>
</tbody>
</table>

Note: (1) Tables 2A-C report the long-run changes following an underlying fiscal shock. For example, if \( x \) is an endogenous variable, we report \( d\tilde{x} = \tilde{x}_1 - \tilde{x}_0 \), where \( \tilde{x}_1 \) is the after-shock steady-state equilibrium value of \( x \) and \( \tilde{x}_0 \) is its pre-shock level.

(2) \( \Psi \) = Long-run (balanced) growth rate. The changes in the growth rates are in percentages.
<table>
<thead>
<tr>
<th></th>
<th>$\sigma = 0$ Private</th>
<th>$\sigma = 0$ Government</th>
<th>$\sigma = 0.5$ Private</th>
<th>$\sigma = 0.5$ Government</th>
<th>$\sigma = 1$ Private</th>
<th>$\sigma = 1$ Government</th>
</tr>
</thead>
<tbody>
<tr>
<td>Subsidy vs Govt Spending (Lump sum tax-financed)</td>
<td>30.35</td>
<td>-5.34</td>
<td>29.92</td>
<td>-1.64</td>
<td>29.57</td>
<td>1.52</td>
</tr>
<tr>
<td>Subsidy vs Govt Spending (Income tax-financed)</td>
<td>33.55</td>
<td>0.73</td>
<td>30.87</td>
<td>2.54</td>
<td>28.65</td>
<td>4.08</td>
</tr>
<tr>
<td>Income Tax Increase</td>
<td>2.96</td>
<td>3.04</td>
<td>0.86</td>
<td>1.08</td>
<td>-0.88</td>
<td>-0.53</td>
</tr>
</tbody>
</table>

Note: Welfare changes are reported as percentages.
Figure 1. Infrastructure Subsidy (Private Provision) vs. Public Spending (Government Provision)

A. Private Provision Model

B. Government Provision Model

i. Capital Utilization Rates

ii. Private Investment-Output Ratio

iii. Consumption-Output Ratio
References


