Financing Public Investment Through Foreign Aid: Consequences for Economic Growth and Welfare

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Abstract

This paper discusses the impact of foreign aid on the growth and macroeconomic performance of a small recipient economy. Both tied and untied aid are considered, as well as permanent and temporary programs. The long-run impact of tied aid and its transitional dynamics depend crucially upon the elasticity of substitution in production, being more effective when this is low. Co-financing arrangements are also emphasized. This contrasts sharply with the effects of untied aid which, with fixed labor supply, has no growth or dynamic consequences but is always welfare improving. These differences also apply to temporary aid flows. Whereas a temporary untied aid shock has only a modest short-run growth effect, its tied counterpart has significant impacts on short-run growth and dynamics. Finally, temporary foreign aid programs, irrespective of their nature, have permanent effects on the levels of key economic variables.

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1. **Introduction**

Aschauer’s (1989a, 1989b) empirical finding that public capital has a powerful impact on the productivity of private capital stimulated a huge volume of both theoretical and empirical research examining the role of public investment on aggregate economic behavior. Notwithstanding the controversy over the validity of Aschauer’s analysis, public investment is now widely accepted as being a crucial determinant of economic growth and macroeconomic performance, although the consensus is that its influence is weaker than originally suggested by Aschauer.¹

One critical issue, especially in poor, resource-constrained developing countries, concerns how new investment in public capital (infrastructure) is to be financed. A significant source for funding public investment in such economies is external financing. This may take the form of borrowing from abroad, through bilateral or multilateral loans, or it may occur through unilateral capital transfers, in the form of tied grants or official development assistance. As Brakman and van Marrewijk (1998) point out, this choice is of some significance. They note that in the post World War II era, capital transfers have increasingly taken the form of development assistance or foreign aid, with between two-thirds and three-fourths of official development assistance being fully or partially tied to investment in infrastructure.² Moreover, the role of tied foreign aid as an instrument for financing investment in public infrastructure has recently assumed greater importance in the context of the expanding European Union. Faced with below average per-capita incomes and low growth rates among some of its joining members, the EU introduced pre-accession aid programs to assist potential member nations in their transition into the union.³ This process of “catching up”

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¹ See Gramlich (1994) for a comprehensive survey of much of the empirical literature. Interest in the impact of public capital on private capital accumulation and economic growth, however, dates back to the seminal work of Arrow and Kurz (1970). The recent growth literature has extended their insights using both the Ramsey model and the ‘AK’ endogenous growth framework; see e.g. Futagami, Morita, and Shibata (1993), Baxter and King (1993), Glomm and Ravikumar (1994), Fisher and Turnovsky (1998). This literature focuses overwhelmingly on closed economies. One exception is Turnovsky (1997) who analyzes the interdependence between private capital and public capital on the growth performance of a small open economy, in many respects structurally similar to that being analyzed in this paper.

² World Bank (1994).

³ Greece, Ireland, Spain, and Portugal were recipients of unilateral capital transfers tied to public investment projects under the Structural Funds Program between 1989-1993 and 1993-1999. A similar tied transfer program, called Agenda 2000, has been initiated for eleven aspiring member nations (Central Eastern European Countries), and is expected to continue until 2006; see European Union (1998a, 1998b).
began in 1989 with a program of unilateral capital transfers from the EU through its Structural Funds program, and subsequent programs were introduced in 1993 and in 2000. These assistance programs tied the capital transfers (or aid) to the accumulation of public capital, and were specifically aimed at building up infrastructure in the recipient nation. The objective of these aid programs was for the recipient economy to correct domestic distortions, attain strong positive growth differentials relative to the union average in the short run, achieve higher and sustainable living standards, and ultimately gain accession to EU membership.

Although the role of foreign aid as an instrument for financing public capital investment is widely acknowledged in the development literature, the standard analysis has been based mainly on static models, and for this reason suffers from several serious weaknesses. First, one of the main purposes of directing the foreign aid to public investment is to stimulate private investment and capital accumulation. This can be studied satisfactorily only in a fully specified dynamic intertemporal framework. Second, most theoretical studies treat foreign aid flows as permanent, while in reality most development assistance programs, such as those initiated in the EU, are only temporary in nature. Third, it is likely that external assistance and borrowing will fail to meet the total financial needs for public investment, and therefore domestic participation by both the recipient government and the private sector is very important. Gang and Khan (1991) and Khan and Hoshino (1992) report that most bilateral aid for public investment in LDCs is tied and is given on the condition that the recipient government invests certain resources into the same project. More recently, in a panel study of 56 developing countries and six four-year periods (1970-93), Burnside and Dollar (2000) find that foreign aid is most effective when combined with a positive policy environment in the recipient economy.

In order to avoid these shortcomings this paper analyzes the effect of foreign aid in an intertemporal model of a small open economy characterized by endogenous growth. In doing so, we explicitly characterize and contrast the dynamic response of such an economy to aid programs that

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4 See Cassen (1986), and more recently, Brakman and van Marrewijk (1998) for a survey of this literature. Two exceptions include Djajic, Lahiri, and Raimondos-Moller (1999), and Hatzipanayotou and Michael (2000), who examine the effects of transfers in an intertemporal context.
may, or may not, be tied to public investment.\(^5\) Both permanent and temporary shocks are considered, and various co-financing arrangements involving participation by the government of the recipient economy are discussed.

The question we address is closely related to the “transfer problem”, one of the classic issues in international economics, dating back to Keynes (1929), Ohlin (1929), and others. This early literature was concerned with “pure” (untied) transfers, which could be in the form of an unrestricted gift or as debt-relief. By contrast, our analysis differs from the existing literature by focusing on “productive” (tied) transfers, the use of which is tied to public investment. Moreover, the formulation we develop parameterizes the transfer so that we can conveniently identify tied and untied aid as polar cases. Embedding the flow of foreign aid (either tied or untied) into an intertemporal framework enables us to compare both its short-run and long-run effects on the dynamic evolution and growth rate of the economy, and ultimately on welfare.

The results we shall be discussing represent an overview of a research program that we have been engaged in over a number of years; see Chatterjee, Sakoulis, and Turnovsky (2002), Chatterjee and Turnovsky (2003a), Chatterjee and Turnovsky (2003b). One of the general conclusions of this research is that the acceptance of a tied transfer by a small growing economy inevitably requires it to undertake some internal structural adjustments, and the flexibility it has to do so will determine the effectiveness (or otherwise) of the transfer program. Thus, the relative merits of tied versus untied transfers on the dynamic evolution of the economy, and their respective desirability from a welfare standpoint, depend crucially on a number of key structural characteristics of the economy that summarize this flexibility. These include: (i) any installation costs associated with the publicly provided capital (intertemporal adjustments); (ii) the degree of access to the world financial market; (iii) the substitutability of public and private capital in production (intratemporal adjustments); (iv) the flexibility of labor supply.

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\(^5\) Bhagwati (1967) points out that tied assistance may take different forms. The transfer or aid from abroad may be linked to a (i) specific investment project, (ii) specific commodity or service, or (iii) to procurement in a specific country. We focus our analysis on the first type of tying, i.e. to an investment project. Examples of such tied capital transfers include the relocation of German capital equipment at the end of the Second World War to Eastern Europe and the Soviet Union, the Marshall Plan in the post-World War II era for the reconstruction of Europe, and more recently, the European Union’s pre-accession aid programs for aspiring member nations.
The remainder of the paper is structured as follows. Section 2 provides an informal overview of the approach we have developed in our previous work and a summary of some of the main results. Section 3 sets out the analytical framework. Most of our research has been conducted numerically and this is discussed in Section 4, which also discusses some significant extensions. Section 5 provides some conclusions, including a suggestion for the direction that further work on this topic might pursue.

2. An Overview and Summary of Main Results

Several common features run throughout our series of studies. First, the economy is small, but faces restricted access to the world capital market. This is expressed in the form of an upward-sloping supply curve of debt, according to which the country’s cost of borrowing depends upon its debt position, relative to its capital stock, the latter serving as a measure of its debt-servicing capability. This assumption is motivated by the large debt burdens of most developing countries, which give rise to the potential risk of default on international borrowing.\(^6\) Second, the production function is linearly homogeneous in public and private capital (the two accumulating assets), so that the economy is capable of sustaining an equilibrium of ongoing growth. Third, new investment in both types of capital is subject to convex costs of installation. This assumption recognizes the fact that factor substitution can occur both intratemporally and/or intertemporally. Fourth, we assume that public investment in infrastructure is financed both by the domestic government, as well as via the flow of international transfers or aid, thereby incorporating the important element of domestic co-financing, characteristic of most bilateral aid programs that are tied to specific public investment projects. Fifth, we parameterize the specification of foreign transfers in such a way that the productive transfer and the pure transfer emerge as polar cases. In all cases, the flow of aid is assumed to be tied to the scale of the recipient economy, and therefore is consistent with maintaining an equilibrium of sustained (endogenous) growth.

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\(^6\) Evidence suggesting that more indebted economies pay a premium on their loans from international capital markets to insure against default risk has been provided by Edwards (1984).
Chatterjee, Sakoulis, and Turnovsky (2002) begin with the simplest assumption, namely that labor supply is fixed, and that the underlying production function is of the Cobb-Douglas form in private and public capital. The main results they obtain include the following.

First, they find that the intertemporal effects of an increase in foreign aid depends critically on whether it is tied or untied. A permanent untied aid program does not generate any dynamic response, but instead leads to instantaneous increases in consumption and welfare. On the other hand, a permanent aid program that is tied to investment in public capital generates transitional dynamics in the recipient economy. The effects of aid were shown to depend upon how the recipient government responds with regard to co-financing. In particular they showed the existence of a sharp tradeoff between co-financing directed at welfare-maximization and growth-maximization, absent in Barro (1990), but present in Futagami et al. (1993).

Second, they examine the tradeoff between the degree of imperfection in the world financial market and the installation costs of public capital. They find that given the former, an increase in the installation costs leads to larger welfare gains from a pure transfer of a given magnitude, but a decrease in welfare gains if the transfer is tied to public capital. For very high installation costs the economy is better-off with a pure transfer: a tied or productive transfer is welfare-reducing in the long run, irrespective of the nature of world capital markets. However, in all other cases, welfare gains from productive transfers are higher than those from pure transfers.

Finally, both a temporary untied aid program and a temporary tied aid program generate transitional dynamics, though of a sharply contrasting nature. Temporary untied aid has only modest short-run growth effects, which impact most directly on private capital and consumption. Tied aid has much more potent short-run growth effects, and by impinging more directly on public capital and debt, yields very different transitional dynamics. By influencing the transitional growth rates, temporary aid flows have *permanent* effects on the *levels* of key variables such as the capital stocks, output, and welfare, these being more significant for the productive transfer.

The assumption of a Cobb-Douglas production function, while a natural benchmark case and prevalent throughout much of the recent endogenous growth literature, is restrictive. Intuitively, one would expect the impact of foreign aid to be sensitive to the degree of intratemporal substitution
between the two types of capital inputs. To analyze this, one needs to employ a more flexible production specification, such as the constant elasticity of substitution (CES) production function, which accommodates alternative degrees of substitution. This task was undertaken by Chatterjee and Turnovsky (2003a). Indeed, their analysis confirms that the elasticity of substitution is a critical determinant of both the dynamic adjustment paths generated by a program of tied-transfers and their welfare implications. The magnitude and the direction of the transitional dynamics and long run effects depend crucially upon the elasticity of substitution between the two types of capital in the recipient economy. The analysis suggests that tied aid is more effective in terms of its impact on long-run growth and welfare for countries that have low substitutability between factors of production. This finding has important policy implications, especially in light of recent empirical evidence suggesting that less developed or poor countries have elasticities of substitution that are significantly below unity. Although Chatterjee and Turnovsky (2003a) restrict their analysis to permanent shocks, the present paper extends that analysis to temporary shocks. One important finding is that the Chatterjee, Sakoulis, and Turnovsky (2002) result that the permanent effects of temporary shocks is more significant for tied transfers, is highly sensitive to the elasticity of substitution in production.

In a more recent paper Chatterjee and Turnovsky (2003b) endogenize the labor supply decision, amending the production function to one in which labor and public capital interact to yield “labor efficiency units” as initially introduced by Romer (1986). This modification too has significant effects. In this case, the pure transfer now generates a dynamic response, although one that contrasts sharply with that of the tied transfer. They also find sharp tradeoffs between the elasticity of substitution in production, and between labor (leisure) and consumption in preferences insofar as the relative welfare effects of tied versus pure transfers are concerned.

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7 In a recent panel study of 82 countries over a 28-year period, Duffy and Papageorgiou (2000) find that they can reject the Cobb-Douglas specification for the entire sample in favor of the more general CES production function. They also report that the degree of substitution between inputs (in their case human and physical capital) may vary with the stages of development. Empirical evidence on the substitutability of public and private capital is sparse. Lynde and Richmond (1993) introduce public and private capital into a more general translog production function for U.K. manufacturing and find that the Cobb-Douglas specification is rejected.
3. **The Analytical Framework**

We now turn to a more formal description of the model, describing its components in turn.\(^8\)

3.1. **Private Sector**

We consider a small open economy populated by an infinitely-lived representative agent who produces and consumes a single traded commodity. Output, \(Y\), of the commodity is produced using the CES production function

\[
Y = \alpha \left[ \eta K^{-\rho} + (1 - \eta) K_G^{-\rho} \right]^{-1/\rho}; \quad \alpha > 0, \quad 0 < \eta < 1, \quad \rho > -1
\]  

(1a)

where \(K\) denotes the representative agent's stock of private capital, \(K_G\) denotes the stock of public capital, and \(\sigma \equiv 1/(1 + \rho)\) is the intratemporal elasticity of substitution between private and public capital in production. We shall abstract from labor so that private capital should be interpreted broadly to include human, as well as physical, capital; see Rebelo (1991).

The agent consumes this good at the rate, \(C\), yielding utility over an infinite horizon represented by the isoelastic utility function:\(^9\)

\[
U = \int_0^\infty \frac{1}{\gamma} C^\gamma e^{-\beta t} dt; \quad -\infty < \gamma < 1
\]

(1b)

The agent also accumulates physical capital, with expenditure on a given change in the capital stock, \(I\), involving adjustment (installation) costs specified by the quadratic (convex) function

\[
\psi(I, K) = I + h_i \frac{I^2}{2K} = I \left(1 + h_i \frac{I}{2K} \right)
\]

(1c)

This equation is an application of the familiar cost of adjustment framework, where we assume that the adjustment costs are proportional to the *rate* of investment per unit of installed capital (rather

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\(^8\) The description of the model follows Chatterjee and Turnovsky (2003a) closely.

\(^9\) The exponent \(\gamma\) is related to the intertemporal elasticity of substitution \(s\), by \(s = 1/(1 - \gamma)\), with \(\gamma = 0\) being equivalent to a logarithmic utility function.
than its level). The linear homogeneity of this function is necessary for a steady-state equilibrium having ongoing growth to be sustained. The net rate of capital accumulation is thus:

$$\dot{K} = I - \delta_K K$$

(1d)

where $\delta_K$ denotes the rate of depreciation of private capital.

Agents may borrow internationally on a world capital market. The factor we wish to take into account is that the creditworthiness of the economy influences its cost of borrowing from abroad. Essentially, we assume that world capital markets assess an economy's ability to service debt costs and the associated default risk, the key indicator of which is the country's debt-capital (equity) ratio. As a result, the interest rate countries are charged on world capital markets increases with this ratio. This leads to the upward sloping supply schedule for debt, expressed by assuming that the borrowing rate, $r(N/K)$, charged on (national) foreign debt, $N$, relative to the stock of private capital, $K$, is of the form:

$$r(N/K) = r^* + \omega(N/K) \quad \omega' > 0$$

(1e)

where $r^*$ is the exogenously given world interest rate, and $\omega(N/K)$ is the country-specific borrowing premium that increases with the nation's debt-capital ratio. The homogeneity of the relationship is required to sustain a balanced growth equilibrium. 11

The agent’s decision problem is to choose consumption, and the rates of accumulation of capital and debt, to maximize intertemporal utility (1b) subject to the flow budget constraint

$$\dot{N} = C + r(N/K)N + \Psi(I, K) - (1 - \tau)Y + \bar{T}$$

(2)

10 Chatterjee, Sakoulis, and Turnovsky (2002) define $\omega$ to be a function of $N/(K + K_g)$, rather than $N/K$. However, this makes very little difference to the results.

11A rigorous derivation of (1e) presumes the existence of risk. Since we do not wish to model a full stochastic economy, we should view (1e) as representing a convenient reduced form, one supported by empirical evidence; see e.g. Edwards (1984) who finds a significant positive relationship between the spread over LIBOR (e.g. $r'$) and the debt-GNP ratio. Eaton and Gersovitz (1989) provide formal justifications for the relationship (1e). Various formulations can be found in the literature. The original formulation by Bardhan (1967) expressed the borrowing premium in terms of the absolute stock of debt; see also Obstfeld (1982), Bhandari, Haque, and Turnovsky (1990). Other authors such as Sachs (1984) also argue for a homogeneous function such as (1e). We have also considered the Edwards (1984) formulation, $r = r(N/Y)$, and very similar results to those reported are obtained.
where $N$ is the stock of debt held by the private sector, $\tau$ is the income tax rate, and $\mathcal{T}$ denotes lump-sum taxes.\footnote{It is natural for us to assume $N > 0$, so that the country is a debtor nation. However, it is possible for $N < 0$ in which case the agent accumulates credit by lending abroad. For simplicity, interest income is assumed to be untaxed.} It is important to emphasize that in performing his optimization, the representative agent takes the borrowing rate, $r(\cdot)$ as given. This is because the interest rate facing the debtor nation, as reflected in its upward sloping supply curve of debt, is a function of the economy's aggregate debt-capital ratio, which the individual agent assumes he is unable to influence.

The optimality conditions with respect to the representative agent’s decision problem can be expressed as follows

\begin{align}
C^{\gamma-1} & = \nu \\
1 + h_i(I/K) & = q \\
\frac{\dot{K}}{K} & = \phi_k = \frac{q-1}{h_i} - \delta_k \\
\beta - \frac{\dot{\nu}}{\nu} & = r\left(\frac{N}{K}\right) \\
\frac{\dot{C}}{C} & = \phi_c = \frac{1}{1-\gamma} \left[r\left(\frac{N}{K}\right) - \beta\right] \\
\frac{(1-\tau)(1-\eta)}{q} \left[\eta(K_G/K)^{-\rho} + (1-\eta)\right]^{-(1+\rho)/\rho} + \frac{\dot{q}}{q} + \frac{(q-1)^2}{2h_iq} - \delta_k & = r\left(\frac{N}{K}\right)
\end{align}

where $\nu$ is the shadow value of wealth in the form of internationally traded bonds, $q'$ is the shadow value of the agent’s private capital stock, and $q \equiv q'/\nu$ is thus the market price of private capital in terms of the (unitary) price of foreign bonds. The interpretation of the above optimality conditions is standard and are discussed by Chatterjee and Turnovsky (2003a). Finally, in order to ensure that the agent’s intertemporal budget constraint is met, the following transversality conditions must hold:

\begin{align}
\lim_{t \to \infty} v Be^{-\beta} & = 0; \ \lim_{t \to \infty} q' Ke^{-\beta} = 0.
\end{align}
3.2 Public Capital, Foreign Aid, and National Debt

The resources for the accumulation of public capital come from two sources: domestically financed government expenditure on public capital, $\bar{G}$, and a program of capital transfers or foreign aid, $TR$, from the rest of the world. We therefore postulate

$$G \equiv \bar{G} + \lambda TR$$

where $\lambda$ represents the degree to which the transfers from abroad are tied to investment in the stock of public infrastructure. The case $\lambda=1$ implies that transfers are completely tied to investment in public capital, representing a “productive” transfer. In the other polar case, $\lambda=0$, incoming transfers are not invested in public capital and hence represent a “pure” transfer, of the Keynes-Ohlin type.

We assume that the gross accumulation of public capital, $G$, is also subject to convex costs of adjustment, similar to that of private capital

$$\Omega(G,K_G) = G\left(1 + \left(h_2/2\right)\left(G/K_G\right)\right).$$

In addition, the stock of public capital depreciates at the rate $\delta_G$ so that the net rate of public capital accumulation is

$$\dot{K}_G = G - \delta_G K_G.$$  \hfill (5)

To sustain an equilibrium of on-going growth, both domestic government expenditure on infrastructure ($\bar{G}$) and the flow of aid from abroad must be tied to the scale of the economy

$$\bar{G} = \bar{g}Y, \text{ and } TR = \theta Y, \text{ } 0 < \bar{g} < 1, \theta > 0, \text{ } 0 < \bar{g} + \theta < 1$$

and

$$G = \bar{G} + \lambda TR = (\bar{g} + \lambda \theta)Y$$

We can therefore rewrite (5) in the following form

$$\dot{K}_G = (\bar{g} + \lambda \theta)Y - \delta_G K_G.$$  \hfill (5’)

\[\text{Noting the definition of } G, \text{ we see that the transfers contribute to the financing of the installation costs, as well as to the accumulation of the new public capital.}\]
and the growth rate of public capital is therefore given by

\[ \frac{\dot{K}_G}{K_G} \equiv \phi_G = (\bar{g} + \lambda \theta) \frac{Y}{K_G} - \delta_G. \] (6)

The government sets its tax and expenditure parameters to continuously maintain a balanced budget:

\[ T + TR + \bar{T} = \Omega(G, K_G) \] (7)

The national budget constraint, (the nation’s current account) can be obtained by combining (7) and (2),

\[ N = r(N/K)N + C + \Psi(I, K) + \Omega(G, K_G) - Y - TR. \] (8)

Equation (8) states that the economy accumulates debt to finance its total expenditures on public capital, private capital, consumption and interest payments net of output produced and transfers received. It is immediately apparent that higher consumption or investment raises the rate at which the economy accumulates debt. The direct effect of a larger unit transfer on the growth rate of debt is given by \( (\lambda - 1) + \left( h_z/K_G \right) \lambda G \). An interesting observation is that the more transfers are tied to public investment (the higher \( \lambda \)), the lower the decrease in the growth rate of debt. When transfers are completely tied to investment in infrastructure, i.e., \( \lambda = 1 \), debt increases due to higher installation costs. However, the indirect effects, induced by the change will still need to be taken into account.

3.3. Macroeconomic Equilibrium

The steady-state equilibrium has the characteristic that all real quantities grow at the same constant rate and that \( q \), the relative price of capital, is constant. Thus we shall express the dynamics of the system in terms of the following stationary variables, normalized by the stock of private capital, \( c \equiv C/K, \ k_g \equiv K_G/K, \ n \equiv N/K, \) and \( q \). The equilibrium system can be described as follows.

\[ \frac{\dot{k}_g}{k_g} \equiv \phi_g - \phi_k = \alpha(\bar{g} + \lambda \theta)[\eta + (1 - \eta)k_g^\rho] - \frac{q - 1}{h_i} - (\delta_G - \delta_K) \] (9a)
\[
\dot{n} = \frac{\phi_n - \phi_\kappa}{n} = r(n) + \frac{1}{n} \left\{ \left( \bar{g} + \lambda \theta \right) - \left( 1 + \theta \right) \right\} y + \frac{q^2 - 1}{2h_1} + \frac{h_2}{2} (\bar{g} + \lambda \theta)^2 \frac{y^2}{k^c} + c \\
- \left( \frac{q - 1}{h_1} \right) + \delta_\kappa
\]  
(9b)

\[
\frac{\dot{c}}{c} = \phi_\kappa - \phi_\kappa = \frac{r(n) - \beta}{1 - \gamma} - \frac{q - 1}{h_1} + \delta_\kappa
\]  
(9c)

\[
\dot{q} = \left[ r(n) + \delta_\kappa \right] q - \alpha(1 - \tau)(1 - \eta) \left( \eta k_g^{-\rho} + (1 - \eta) \right)^{(1 + \rho)\rho} - \frac{(q - 1)^2}{2h_1}
\]  
(9d)

Equations (9a) – (9d) provide an autonomous set of dynamic equations in \( k, n, c, \) and \( q, \) from which the steady-state equilibrium and transitional dynamics can be derived.

3.4. The Steady-State Equilibrium and Transitional Dynamics

The economy reaches steady-state equilibrium when \( \dot{k} = \dot{n} = \dot{c} = \dot{q} = 0, \) implying that \( \dot{K}/K = \dot{K}_G/K_G = \dot{N}/N = \dot{C}/C = \dot{\phi}, \) the steady-state growth rate of the economy. Applying the above conditions in (9a)-(9d) we can determine \( \ddot{k}, \ddot{q}, \ddot{n}, \) and \( \ddot{c}, \) along with the steady-state interest rate \( \ddot{r}(\ddot{n}), \) and the steady-state growth rate \( \ddot{\phi}. \) Because this system is highly non-linear, it need not be consistent with a well-defined steady-state equilibrium with \( \ddot{k} > 0, \ddot{c} > 0. \) Our numerical simulations, however, yield well-defined steady-state values for all plausible specifications of the structural and policy parameters of the model.\(^{14}\)

Linearizing equations (9a)–(9d) around the steady-state values of \( k, n, q, \) and \( c, \) the dynamics can be approximated by a linear system of the form

\[
\dot{\tilde{X}} = \Lambda \left( \tilde{X} - \tilde{X} \right)
\]  
(11)

where \( \tilde{X}' \equiv (k, n, c, q), \) \( \tilde{X}' \equiv (\ddot{k}, \ddot{n}, \ddot{c}, \ddot{q}), \) and \( \Lambda \) represents the coefficient matrix of the linearized system.

\(^{14}\)A discussion of issues pertaining to non-existent or multiple equilibria in a related model is provided by Turnovsky (2000). Similar issues apply here.
The determinant of the coefficient matrix, $\Lambda$, of (11) can be shown to be positive under the condition that $\bar{r}(\bar{n}) > \bar{\phi}$ i.e., the steady-state interest rate facing the small open economy must be greater than the steady-state growth rate of the economy. Imposing the transversality condition (4c), we see that this condition is indeed satisfied. Since (11) is a fourth-order system, a positive determinant implies that there could be 0, 2, or 4 positive (unstable) roots. However, our numerical simulations yield saddle-point behavior for all plausible ranges of parameters. Thus we shall assume that the dynamic system (11) is saddle-point stable, having two positive (unstable) and two negative (stable) roots, the latter being denoted by $\mu_1$ and $\mu_2$, with $\mu_2 < \mu_1 < 0$.

4. The Dynamic Effects of Foreign Aid: A Numerical Analysis

Due to the complexity of the model, we will employ numerical methods to examine the dynamic effects of a foreign aid or a transfer shock. We begin by calibrating a benchmark economy, using the following parameters representative of a small open economy, which starts out from an equilibrium with zero transfers/aid.\(^{15}\)

**The Benchmark Economy**

<table>
<thead>
<tr>
<th>Category</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Preference parameters:</td>
<td>$\gamma = -1.5, \beta = 0.04$</td>
</tr>
<tr>
<td>Production parameters:</td>
<td>$\alpha = 0.4, \eta = 0.2, h_1 = 15, h_2 = 15$</td>
</tr>
<tr>
<td>Elasticity of substitution in production:</td>
<td>$\sigma = 0.33, 1, \to \infty$</td>
</tr>
<tr>
<td>Depreciation rates:</td>
<td>$\delta_K = 0.05, \delta_G = 0.04$</td>
</tr>
<tr>
<td>World interest rate:</td>
<td>$\bar{r} = 0.06$</td>
</tr>
<tr>
<td>Premium on borrowing:</td>
<td>$a = 0.1^{16}$</td>
</tr>
<tr>
<td>Policy parameters:</td>
<td>$\tau = 0.15, \bar{g} = 0.05$</td>
</tr>
<tr>
<td>Transfers:</td>
<td>$\theta = 0, \lambda = 0$</td>
</tr>
</tbody>
</table>

The critical parameter upon which we focus is the elasticity of substitution, $\sigma$, and we consider three benchmark economies, depending on the degree of substitutability between public and

\(^{15}\) The empirical justification for these parameters is discussed at greater length by Chatterjee and Turnovsky (2003a).

\(^{16}\) The functional specification of the upward sloping supply curve that we use is: $r(n) = \bar{r} + e^{am} - 1$. Thus, in the case of a perfect world capital market, when $a = 0$, $r = \bar{r}$, the world interest rate.
private capital in production. These include: (i) low elasticity of substitution, $\sigma = 0.33$ (Table 1A); (ii) Cobb-Douglas production function, with unitary elasticity of substitution, $\sigma = 1$ (Table 1B), and (iii) perfect substitutability between the two types of capital, where $\sigma \to \infty$ (Table 1C).

The calibrated benchmark economies derived from the above parameter specifications are reported in Table 1. The standard case of the Cobb-Douglas specification is reported in Table 1B, Row 1. It implies a steady-state ratio of public to private capital of 0.29; the consumption-output ratio is 0.60, the debt to GDP ratio of 0.45, leading to an equilibrium borrowing premium of 1.42% over the world rate of 6%. The capital-output ratio is over 3, with the equilibrium growth rate being around 1.37%. This equilibrium is reasonably characterizes a small-medium indebted economy, experiencing a modest steady rate of growth and having a relatively small stock of public capital.

Parts 1A-1C reveal the sensitivity of the steady-state equilibrium to variations in the elasticity of substitution in production. For a very low degree of substitution in production, $\sigma = 0.33$ (Benchmark I, Table 1A, Row 1), the steady-state ratio of public to private capital is increased to 0.437, the interest rate is 2.4%, lower than the world interest rate of 6%, which implies that this economy is a net creditor to the rest of the world, and thus has an initial current account surplus. This is reflected in a debt-output ratio of −1.24. The low elasticity of substitution causes agents to lower their investment in the stock of private capital, and enjoy higher consumption, leading to a consumption-output ratio of 0.78. Due to the low investment in private capital and high consumption, the steady-state growth rate in this economy is −0.6%.

In the extreme case of perfect substitutability between public and private capital (Benchmark III, Table 1C, Row 1), the equilibrium ratio of public to private capital decreases to 0.27. The consumption-output ratio decreases to 0.51 and the current account deficit increases, reflected in a higher debt-GDP ratio of 1.11 and a steady-state interest rate of 9.87%. The high elasticity of substitution leads to an equilibrium growth rate of 2.35%.

4.1. Permanent Foreign Aid Shock: Long Run Effects

We now consider a permanent increase in foreign aid flows to the above benchmark specifications. Specifically, the foreign aid is tied to the scale of the economy, and increases from
0% of GDP in the initial steady-state to 5% of GDP in the new steady-state (an increase in $\theta$ from 0 to 0.05). However, this aid may be tied to new investment in public capital ($\lambda = 1$), representing the case of a “productive” transfer, or it may be untied ($\lambda = 0$), representing the case of a “pure” transfer from abroad. The short-run and long-run responses of key variables in the recipient economy are reported in Rows 2 and 3 in Tables 1A - 1C, which correspond to the varying elasticity of substitution. The final column in the table summarizes the effects on economic welfare, measured by the optimized utility of the representative agent

$$W = \int_0^\infty \frac{1}{\gamma} C^\gamma e^{-\beta t} dt$$

where $C$ is evaluated along the equilibrium path. These welfare changes are calculated as the percentage change in the initial stock of capital necessary to maintain the level of welfare unchanged following the particular shock. We first discuss the long-run effects of the foreign aid shock (Table1) and then proceed to consider the transitional dynamics generated by this shock (Fig. 1).

4.1.1 Tied Aid

The long-run impact of a tied aid shock is reported in Rows 2 of Tables 1A-1C. Since the aid is tied to new investment in public capital, the implied long-run increase in the stock of public capital increases the long-run marginal product of private capital and generates a dynamic adjustment for its market price, $q$. However, the magnitude and direction of the initial response of $q$ and its consequent dynamic adjustment will depend crucially on the elasticity of substitution between the two types of capital stocks, $\sigma$.

Row 2 of Table 1B describes the standard case of the Cobb-Douglas production function. In the new steady state the ratio of public to private capital increases from 0.29 to 0.61, thereby generating a huge investment boom in infrastructure. The increase in the stock of public capital increases the marginal productivity of private capital, thereby leading to a positive, though lesser accumulation of private capital. Although the transfer stimulates consumption through the wealth effect, (like the pure transfer) the higher long-run productive capacity has a greater effect on output,
leading to a decline in the long-run consumption-output ratio from 0.60 to 0.56. The higher productivity raises the long-run growth rate to 1.94 %, while long run welfare improves by 9.83%, as indicated in the last column of Row 3. The increased accumulation of both private and public capital lead to a higher demand for external borrowing as a means of financing new investment in private capital and the installation costs of public capital. This results in an increase in the steady-state debt-output ratio from 0.45 to 0.77, raising the borrowing premium to over 2.8%. However, this higher debt relative to output is sustainable since it is caused by higher investment demand rather than higher consumption demand. The long-run increase in the economy’s productive capacity (as measured by the higher stocks of public and private capital, and output) ensures that the higher debt is sustainable. This view has also been expressed by Roubini and Wachtel (1998).

The dependence of the efficacy of a tied aid program on the elasticity of substitution in production in the recipient country is an important question. Some indication of this is provided in the three panels of Table 1. We find that as $\sigma$ increases, a tied aid program leads to generally higher long-run increases in the ratio of public to private capital and smaller increases in $q$. Due to the smaller increases in $q$ as $\sigma$ increases, the increase in private investment is also reduced. This leads to less borrowing, reflected by a decline in the increase in the equilibrium interest rate and debt-GDP ratio as $\sigma$ increases. In fact, as $\sigma$ approaches infinity, the current account actually improves. The crowding out of consumption also declines as $\sigma$ increases, thereby reflecting lower induced private investment due to higher substitutability in production. An increase in $\sigma$ also reduces the positive effect of the tied aid program on growth and welfare: the long run gains from both growth and welfare decline as substitutability in production goes up. In the limiting case of perfect substitutability, the welfare gains from untied aid dominate the losses from tied aid.

The above results lead us to believe that insofar as its effect on long run growth and welfare is concerned, a tied aid program is more effective in countries having a low elasticity of substitution in production. This observation complements the recent findings of Duffy and Papageorgiou (2000) that less developed or poor countries have elasticities of substitution that are significantly below unity and developed or richer countries have elasticities that are significantly above unity. In such a
scenario, our analysis shows that a tied aid program may be more effective for poor countries than for their richer counterparts.

Chatterjee and Turnovsky (2003a) conduct a sensitivity analysis to show that the effects of the tied transfer are highly sensitive to relatively minor changes in $\sigma$ from this benchmark value. Thus, for example, if a researcher obtains an empirical estimate for the elasticity of substitution of $\sigma = 1$ with a standard error of 0.1 -- a tight estimate -- then with 95% probability the implied estimated welfare gain of 9.83% could be as high as 14.8% or as low as 7%.

4.1.3. Transitional Dynamics

We first analyze the dynamic effects of a permanent tied aid shock for our benchmark specification of unitary elasticity of substitution between the two types of capital (Cobb Douglas). The transitional adjustment paths are depicted in Fig. 1. Fig. 1.1 illustrates the stable adjustment-locus in $k_g$-$n$ space, indicating how $k_g$ and $n$ increase almost proportionately during the transition. Figs. 1.2-1.5 illustrate the transition paths for the shadow price of private capital, the consumption-capital ratio, the consumption-output ratio, and the debt-output ratio, respectively. The implied long-run increase in the stock of public capital following the aid shock causes an instantaneous upward jump in $q$, the shadow price of private capital, to induce a corresponding increase in private investment. Thereafter, as both types of capital are being accumulated, $q$ gradually increases to its new long-run equilibrium. The time paths for the consumption-capital ratio and consumption output ratio are depicted in Figs. 1.3 and 1.4 respectively. There is an initial upward jump in consumption due to the wealth effect created by the initial upward jump in $q$. Thereafter, as private capital accumulation and output increases, the consumption-capital ratio and consumption-output ratios monotonically approach their respective long run equilibrium values. The debt-output ratio increases monotonically through time. This is because the accumulation of public capital raises the average productivity of private capital, while the accumulation of both types of capital raises the need to borrow from abroad. By contrast, the $C/Y$ ratio initially increases before declining through time.

17 The range for $\sigma$ between 0.8 and 1.2 is generally consistent with reconciliation of the alternative estimates of the elasticity of substitution produced during the 1960s and 1970s, provided by Berndt (1976).
This is because the wealth effect associated with the transfer raises consumption immediately, while the effect on the economy’s productive capacity, through capital accumulation, takes time. The contrasting transitional paths of the four growth rates $\phi_K, \phi_C, \phi_T$ and $\phi_c$ toward their common long-run growth rate are shown in Fig. 1.6. The growth rate of output is an average of the growth rates of the two capital stocks. Finally, the growth rate of consumption is unaffected on impact and responds only gradually. The reason for this is evident from (4a’) and the fact that it depends upon the sluggishly evolving debt-capital ratio, $n$.

The nature of the dynamic adjustment also depends on the elasticity of substitution in production. As the elasticity of substitution increases, the curvature of the adjustment path increases.\(^{18}\) The higher the degree of substitution between the two types of capital, the more the transfer increases the initial growth rate of public capital relative to that of private capital. At the same time, the rate of debt accumulation increases raising borrowing costs. Over time, as the growth rate of public capital declines and that of private capital increases, foreign borrowing and borrowing costs fall. For a very high elasticity of substitution, we get very rapidly increasing debt and borrowing costs during the early phases of the transition. However, over time, these inhibit borrowing, which declines and in the limiting case where the two types of capital are perfect substitutes, $n$ ultimately returns to its initial level.

4.1.4. Untied Aid

A permanent pure transfer shock, i.e., an aid flow not tied to any investment activity, does not generate any transitional adjustment and nor does it have any long-run effects on the key variables in the economy except consumption and welfare (Tables 1A-1C, Row 3). The pure transfer immediately raises consumption by the equivalent amount and long-run welfare increases instantaneously as well. For example, for benchmark I, the consumption-output ratio goes up from 0.78 to 0.83, and from 0.60 to 0.65 for benchmark II. For benchmark III, it goes up from 0.51 to 0.56. However, even though long run welfare increases due to an untied aid flow, the gains increase

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\(^{18}\) This case is not illustrated here, but the details are available from the authors upon request.
with the elasticity of substitution. For benchmark I, the gain in welfare is 6.4%, while it is 8.3% and 9.8% for benchmarks II and III respectively.

Whether a permanent untied aid shock will generate transitional dynamics or not depends crucially on the specification of the leisure-labor choice. In case we have been discussing, we have assumed that labor supply is inelastic, so that the entire effect of an untied aid shock is absorbed by consumption. But in general, if the labor supply is endogenous, the marginal rate of substitution between consumption and work effort would be affected, and that in turn, would lead to a dynamic adjustment; see Chatterjee and Turnovsky (2003b).

4.1.5. Co-financing and Welfare Gains

Several aid programs call for co-financing by the domestic government. In Table 2 we compare the welfare effects of the tied and pure aid programs with two alternative forms of co-financing. In the first, the government receives a tied aid flow of 2.5% of its income, which it must match with an equal increase in its expenditure; in the second it must match an untied aid flow. In all four cases, the economy is experiencing a 5% increase in expenditure.

For low or medium elasticity of substitution the tied transfer (TT) is superior to the pure transfer (PT), where as for a high $\sigma$ this ordering is reversed, as we have seen. In all cases the matched tied transfer (MTT) is dominated by TT. This is because the MTT involves making the size of the government sector too large. While the matched pure transfer (MPT) is never dominant, it is superior to PT in the case where $\sigma = 0.33$ and it is superior to TT as $\sigma \rightarrow \infty$.19

We can also show that a tied aid of a given amount, coupled with an equivalent decrease in domestic government expenditure, is equivalent to an untied transfer of an equivalent amount. This is important since it implies that by combining the transfer with the appropriate expenditure and tax mix, the recipient economy can choose an equilibrium path and associated level of welfare that is independent of any constraints imposed by the donor country.

Numerical simulations conducted by Chatterjee, Sakoulis, and Turnovsky (2002) also

19 Chatterjee, Sakoulis, and Turnovsky (2002) address the question of optimal co-financing in the case of the Cobb-Douglas production function. The analogous exercise can be pursued here.
illustrate a dramatic tradeoff between growth maximization and welfare maximization. Growth maximization implies a greater participation of the government in the economy through its spending on public capital. This emphasis on growth and capital accumulation leaves less output available for consumption, which is undesirable from a welfare standpoint. This tradeoff between the two policy objectives is absent from Barro (1990), but present in Futagami et al. (1993).

4.2. Temporary Foreign Aid Shock

Most foreign aid programs, whether tied or untied, are only temporary. Thus it is important to analyze the consequences of a temporary transfer. We assume that the duration of the transfer is 10 years, consistent with the average length of the EU’s Structural Funds Programs. The results of our experiments are illustrated in Figs. 2 and 3. As before, the graphical illustrations correspond to the Cobb-Douglas case of unitary elasticity of substitution.

4.2.1. Untied Aid

We turn first to the case of a temporary untied aid shock that is in effect for a period of ten years. Neither the growth rate of consumption nor debt responds immediately. In the case of consumption, the reason for this remains the same as for the permanent transfer: its growth rate is tied via the borrowing rate to the debt-capital ratio, $n$, which is constrained to evolve continuously over time. Similarly, when $\lambda = 0$, (5') implies that the growth rate of public capital responds only to the productivity of public capital, $Y/K_G$, which evolves gradually. By contrast, the growth rate of private capital, being determined by $q$, does respond on impact. This is because with the transfer being only temporary, the initial response in the consumption-output ratio is dampened, thereby freeing some domestic output, which then becomes available for investment in private capital.

In contrast to a permanent untied aid shock, the adjustments are characterized by transitional dynamics. These can be understood by considering Fig. 2.1, the phase diagram describing the dynamic adjustments of the ratio of public to private capital, $k_g$, and the debt-capital ratio $n$. Suppose that the economy starts out from the equilibrium point A in Fig. 2.1. Since the transfer has

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20 The formal solutions for the case of temporary shocks are provided in an Appendix available from the authors.
no impact on the initial growth of public capital, but leads to more private investment, the ratio of public capital to private capital, $k_g$, begins to decline. At the same time, while the untied transfers reduce the accumulation of debt, the higher investment and consumption has the opposite effect. On balance, the former effect dominates, and the initial growth rate of debt falls from its benchmark value of 1.37% to 0.5%, so that the debt-capital ratio, $n$, begins to decline as well. The economy therefore begins to move along the locus AB in Fig 2.1. After 10 periods, when the transfer ceases, the economy is at B. However, the end of the aid program immediately raises the growth rate of debt, so that the debt-capital ratio starts to increase. By contrast, with private capital still being accumulated at a faster rate than public capital, $k_g$ continues to decline, though with the former declining and the latter increasing, this decline ceases at time 15, when the economy is at C. Thereafter, the reduced relative stock of public capital raises its productivity, encouraging public investment so that the economy returns to its original equilibrium along CA, with both $k_g$ and $n$ increasing. Figs. 2.2 - 2.5 mirror the above dynamic adjustment by illustrating the dynamic time paths of the market price of private capital, the consumption-capital and consumption-output, and debt-output ratios, respectively. Fig. 2.6 depicts the differential dynamic adjustment of the growth rates of key variables: the growth rate of output is seen to be an average of that of the two capital stocks, while the time path for the consumption growth rate reflects that of the time path of $n$.

4.2.2. Tied Aid

Fig. 3 reports the impact of a temporary tied (productive) aid shock. Again, the growth rates of all variables except consumption respond instantaneously. It is interesting to observe that when compared to the corresponding jumps for a permanent tied aid shock, the temporary tied aid induces a marginally larger initial responses in growth rates than does a permanent shock of equal magnitude. Thus, in the short run, while the transfer program is in effect, strong positive differentials are created in growth rates relative to the benchmark.

The dynamics indicate a dramatic contrast from those of the pure transfer; indeed the time paths for most variables are generally reversed. Suppose that the economy starts out at point A in Fig.3.1. With the increase in the growth rate of public capital, far exceeding that of private capital,
the ratio of public to private capital begins to rise. At the same time, with the tied transfers being unavailable for debt reduction, the higher consumption and investment leads to a similar increase in the growth rate of debt, which increases at the rate of 19% on impact, so that the debt-capital ratio begins to rise sharply as well. The economy therefore begins to move along the locus ACB in Fig. 3.1. As \(k_g\) and \(n\) both increase, the growth rates of both public capital and debt decline dramatically, the latter more so, with the economy reaching B after 10 periods. The permanent elimination of the transfer at that time reverses the dynamics, taking the economy back to its original equilibrium along the locus BA.

Figs. 3.2-3.5 present the dynamic paths of the price of private capital, consumption-capital, consumption-output and debt-output ratios respectively. The dynamics of the debt-output ratio and the consumption-output ratio are in contrast with those for the untied aid case, reflecting the reversal in the dynamics of \(k_g\) and \(n\). One interesting difference arises with respect to the consumption-output ratio, which falls below its benchmark during the period the transfer is in effect. This is due to a short run substitution of consumption for capital accumulation. However, the end of the transfer program causes a reverse substitution towards consumption, and the ratio increases to its benchmark in the long run. The main general picture that emerges in comparing Figs. 2 and 3, is that the particular nature of the incoming aid flow has important implications for the economy’s dynamic adjustment, both in the short run as well as in the long run. In our case, the transitional dynamics of an untied aid program are very different from those of a tied aid program.

4.2.3. Permanent Effects of Temporary Aid Programs

In this subsection we show how a temporary foreign aid program, by altering the growth rates during the transition, can have permanent effects on the levels of key variables, such as the capital stock, output, and consumption of the recipient economy. In addition we show how the type of incoming aid (untied or tied) and the elasticity of substitution in production affects the magnitude and direction of the permanent effects. Table 3 reports the permanent effects of temporary aid programs and their sensitivity to the underlying production elasticity of substitution. Specifically, we normalize the benchmark steady state level to unity and express the new steady state levels
relative to the normalized benchmark. For example, the ratio of 1.10 across steady states implies a 10% increase in levels relative to the benchmark. However, the welfare calculations (last column in table 3) are reported as percentage changes.

From Table 3 we see that temporary transfers do indeed have permanent effects on the levels of key economic variables. However, as the results reveal, the magnitude of the effects are different, depending upon the specific nature of the aid program and the elasticity of substitution between the two types of capital. For example, we see that when the elasticity of substitution is low (\(\sigma = 0.33\)) a temporary untied aid shock leads to only a 3% long run improvement in the stock of private capital and a 2% improvement in the stock of public capital and in the levels of consumption and output. The debt position of the economy improves by 12% in the long run. On the other hand, the long run effects of tied aid are less uniform and much larger in magnitude. The long run stocks of private and public capital increase by 30% and 35% respectively. Consumption and output increase by 30% and 33% respectively, while the current account experiences a significant improvement of 89%, much larger than in the case of untied aid. This is due to the fact that the increase in long run productive capacity, as measured by the long run changes in the stock of private and public capital, and the level of output, is much larger under a tied aid program. The higher long run productive capacity enables the economy to improve its long-run debt position significantly. The effects on intertemporal welfare are also substantial under tied aid, in the order of 29.18%, while untied aid leads to a modest long-run welfare increase of 2.79%.

However, as the elasticity of substitution increases, the long-run effects of tied aid decline in magnitude. For example, when the two types of capital are perfectly substitutable, a tied aid shock worsens the long-run capital stocks, consumption and output by about 1%, while under untied aid, they improve by 5%. Welfare worsens by 0.18% under tied aid while untied aid improves it by 6.43%. As the elasticity of substitution increases, the long-run welfare benefits from tied aid decline, while those from untied aid increase. From a welfare point of view, untied aid dominates tied aid for high degrees of substitution in production. The current account also worsens with increases in substitutability for both types of aid. However, the debt position worsens much more

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21 The formal details are provided in an appendix available on request from the authors.
under untied aid relative to tied aid, irrespective of the elasticity of substitution.

5. Conclusions

In this paper we have provided an overview of a body of research we have conducted on an important issue, namely the impact of a foreign aid program on the growth and macroeconomic performance of a small recipient economy. We have considered both tied and untied aid, as well as the efficacy of permanent and temporary aid programs, the former serving as a benchmark, the latter being a closer representation of actual policies.

We find that the long-run impact of a tied aid program and the nature of the transitional dynamics it generates depend crucially upon the elasticity of substitution in production. Our numerical simulations suggest that tied aid is more effective in economies with a low degree of substitution between factors of production. There is a sharp contrast in the effects of untied aid of the traditional Keynes-Ohlin type and the type of tied aid programs recently adopted by the European Union. With fixed labor supply, a permanent untied aid has no growth or dynamic consequences but is always welfare improving. On the other hand, tied aid generates dynamic adjustments, as public capital is accumulated in the recipient economy. Its effect on the long-run growth rate, and the extent to which this is beneficial, depends upon the elasticity of substitution in production, as well as the co-financing arrangements, if any, imposed on the recipient economy, and how its government chooses to react to the additional flow of resources.

These contrasts also apply to temporary aid flows, and in particular to the transitional dynamics in the two cases. Whereas a temporary untied aid shock has only a modest short-run growth effect, its tied counterpart has significant impacts on short-run growth and dynamics. Temporary foreign aid programs, irrespective of their nature, have permanent effects on levels of key economic variables. However, the magnitude and direction of these long-run changes depend crucially on the degree of substitutability of capital. The higher the substitutability in production, the stronger is the case for untied aid. These findings imply that when donors decide on whether to tie a particular aid program to public investment, careful attention should be paid to the recipient’s opportunities for substitution in production, the particular co-financing arrangements that are
imposed with the aid, and the duration for which the aid program will be in effect. Failure to take these factors into consideration may have unintended adverse effects on the recipient economy.

In concluding, we should note that we restricted our discussion to the effects of the transfer on the economic performance of a small recipient economy. Being small, this has no feedback on the donor economy. However, in the case of the EU, for example, such transfers are being proposed simultaneously for a number of prospective member nations, the collective feedback effects of which on the donor economy need no longer be negligible. Thus, an important extension of this analysis is to consider the impact of the transfer in a multi-country growth equilibrium setting.
### TABLE 1: Responses to a Permanent Transfer Shock

#### 1A. Benchmark I

Low Substitutability in Production: $\sigma = 0.33$

<table>
<thead>
<tr>
<th>Benchmark I</th>
<th>$\tilde{k}_g$</th>
<th>$% \tilde{r}$</th>
<th>$\sim C/Y$</th>
<th>$\sim N/Y$</th>
<th>$\phi_K(0)$</th>
<th>$\phi_G(0)$</th>
<th>$\phi_Y(0)$</th>
<th>$\phi_C(0)$</th>
<th>$\sim \phi$</th>
<th>$\Delta(W)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\theta = 0, \lambda = 0,$ $\bar{g} = 0.05, \tau = 0.15$</td>
<td>0.437</td>
<td>2.42</td>
<td>0.778</td>
<td>-1.24</td>
<td>-0.60</td>
<td>-0.60</td>
<td>-0.60</td>
<td>-0.60</td>
<td>-0.60</td>
<td>---</td>
</tr>
<tr>
<td>Tied transfer</td>
<td>$\theta = 0.05, \lambda = 1,$ $\bar{g} = 0.05, \tau = 0.15$</td>
<td>0.696</td>
<td>7.04</td>
<td>0.643</td>
<td>0.29</td>
<td>1.04</td>
<td>2.73</td>
<td>1.99</td>
<td>-0.60</td>
<td>1.22</td>
</tr>
<tr>
<td>Pure-transfer</td>
<td>$\theta = 0.05, \lambda = 0,$ $\bar{g} = 0.05, \tau = 0.15$</td>
<td>0.437</td>
<td>2.42</td>
<td>0.828</td>
<td>-1.24</td>
<td>-0.6</td>
<td>-0.60</td>
<td>-0.60</td>
<td>-0.60</td>
<td>-0.60</td>
</tr>
</tbody>
</table>

#### 1B. Benchmark II

Unitary Substitutability in Production: $\sigma = 1$

<table>
<thead>
<tr>
<th>Benchmark II</th>
<th>$\tilde{k}_g$</th>
<th>$% \tilde{r}$</th>
<th>$\sim C/Y$</th>
<th>$\sim N/Y$</th>
<th>$\phi_K(0)$</th>
<th>$\phi_G(0)$</th>
<th>$\phi_Y(0)$</th>
<th>$\phi_C(0)$</th>
<th>$\sim \phi$</th>
<th>$\Delta(W)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\theta = 0, \lambda = 0,$ $\bar{g} = 0.05, \tau = 0.15$</td>
<td>0.291</td>
<td>7.42</td>
<td>0.60</td>
<td>0.45</td>
<td>1.37</td>
<td>1.37</td>
<td>1.37</td>
<td>1.37</td>
<td>1.37</td>
<td>---</td>
</tr>
<tr>
<td>Tied transfer</td>
<td>$\theta = 0.05, \lambda = 1,$ $\bar{g} = 0.05, \tau = 0.15$</td>
<td>0.61</td>
<td>8.84</td>
<td>0.561</td>
<td>0.774</td>
<td>1.70</td>
<td>6.74</td>
<td>2.71</td>
<td>1.37</td>
<td>1.938</td>
</tr>
<tr>
<td>Pure-transfer</td>
<td>$\theta = 0.05, \lambda = 0,$ $\bar{g} = 0.05, \tau = 0.15$</td>
<td>0.291</td>
<td>7.42</td>
<td>0.651</td>
<td>0.45</td>
<td>1.37</td>
<td>1.37</td>
<td>1.37</td>
<td>1.37</td>
<td>1.37</td>
</tr>
</tbody>
</table>

#### 1C. Benchmark III

Perfect Substitutability in Production: $\sigma \equiv \infty$

<table>
<thead>
<tr>
<th>Benchmark III</th>
<th>$\tilde{k}_g$</th>
<th>$% \tilde{r}$</th>
<th>$\sim C/Y$</th>
<th>$\sim N/Y$</th>
<th>$\phi_K(0)$</th>
<th>$\phi_G(0)$</th>
<th>$\phi_Y(0)$</th>
<th>$\phi_C(0)$</th>
<th>$\sim \phi$</th>
<th>$\Delta(W)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\theta = 0, \lambda = 0,$ $\bar{g} = 0.05, \tau = 0.15$</td>
<td>0.269</td>
<td>9.87</td>
<td>0.509</td>
<td>1.11</td>
<td>2.35</td>
<td>2.35</td>
<td>2.35</td>
<td>2.35</td>
<td>2.35</td>
<td>---</td>
</tr>
<tr>
<td>Tied transfer</td>
<td>$\theta = 0.05, \lambda = 1,$ $\bar{g} = 0.05, \tau = 0.15$</td>
<td>0.577</td>
<td>9.87</td>
<td>0.513</td>
<td>1.04</td>
<td>2.08</td>
<td>8.69</td>
<td>2.50</td>
<td>2.35</td>
<td>2.35</td>
</tr>
<tr>
<td>Pure-transfer</td>
<td>$\theta = 0.05, \lambda = 0,$ $\bar{g} = 0.05, \tau = 0.15$</td>
<td>0.269</td>
<td>9.87</td>
<td>0.559</td>
<td>1.11</td>
<td>2.35</td>
<td>2.35</td>
<td>2.35</td>
<td>2.35</td>
<td>2.35</td>
</tr>
</tbody>
</table>
### TABLE 2: Co-financing Tradeoffs and Welfare Gains

#### 2A. Low Substitutability in Production: $\sigma = 0.33$

<table>
<thead>
<tr>
<th>Method</th>
<th>$% \Delta(W)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Tied transfer (TT)</td>
<td></td>
</tr>
<tr>
<td>$\theta = 0.05, \lambda = 1, \overline{g} = 0.05$</td>
<td>50.05</td>
</tr>
<tr>
<td>Pure transfer (PT)</td>
<td></td>
</tr>
<tr>
<td>$\theta = 0.05, \lambda = 0, \overline{g} = 0.05$</td>
<td>6.42</td>
</tr>
<tr>
<td>Matched Tied transfer (MTT)</td>
<td></td>
</tr>
<tr>
<td>$\theta = 0.025, \lambda = 1, \overline{g} = 0.075$</td>
<td>44.90</td>
</tr>
<tr>
<td>Matched Pure transfer (MPT)</td>
<td></td>
</tr>
<tr>
<td>$\theta = 0.025, \lambda = 0, \overline{g} = 0.075$</td>
<td>26.82</td>
</tr>
</tbody>
</table>

**TT > MTT > MPT > PT**

#### 2B. Unitary Substitutability in Production: $\sigma = 1$

<table>
<thead>
<tr>
<th>Method</th>
<th>$% \Delta(W)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Tied transfer (TT)</td>
<td></td>
</tr>
<tr>
<td>$\theta = 0.05, \lambda = 1, \overline{g} = 0.05$</td>
<td>9.83</td>
</tr>
<tr>
<td>Pure transfer (PT)</td>
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<tr>
<td>$\theta = 0.05, \lambda = 0, \overline{g} = 0.05$</td>
<td>8.32</td>
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<tr>
<td>Matched Tied transfer (MTT)</td>
<td></td>
</tr>
<tr>
<td>$\theta = 0.025, \lambda = 1, \overline{g} = 0.075$</td>
<td>5.07</td>
</tr>
<tr>
<td>Matched Pure transfer (MPT)</td>
<td></td>
</tr>
<tr>
<td>$\theta = 0.025, \lambda = 0, \overline{g} = 0.075$</td>
<td>5.35</td>
</tr>
</tbody>
</table>

**TT > PT > MPT > MTT**

#### 2C. Perfect Substitutability in Production: $\sigma \equiv \infty$

<table>
<thead>
<tr>
<th>Method</th>
<th>$% \Delta(W)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Tied transfer (TT)</td>
<td></td>
</tr>
<tr>
<td>$\theta = 0.05, \lambda = 1, \overline{g} = 0.05$</td>
<td>-2.42</td>
</tr>
<tr>
<td>Pure transfer (PT)</td>
<td></td>
</tr>
<tr>
<td>$\theta = 0.05, \lambda = 0, \overline{g} = 0.05$</td>
<td>9.82</td>
</tr>
<tr>
<td>Matched Tied transfer (MTT)</td>
<td></td>
</tr>
<tr>
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<td>-7.34</td>
</tr>
<tr>
<td>Matched Pure transfer (MPT)</td>
<td></td>
</tr>
<tr>
<td>$\theta = 0.025, \lambda = 0, \overline{g} = 0.075$</td>
<td>-1.27</td>
</tr>
</tbody>
</table>

**PT > MPT > TT > MTT**
TABLE 3

Permanent Effects of Temporary Foreign Aid Shocks

<table>
<thead>
<tr>
<th></th>
<th>$K$</th>
<th>$K_g$</th>
<th>$C$</th>
<th>$N$</th>
<th>$Y$</th>
<th>$\Delta W (%)$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$\lambda = 0$</td>
<td>$\lambda = 1$</td>
<td>$\lambda = 0$</td>
<td>$\lambda = 1$</td>
<td>$\lambda = 0$</td>
<td>$\lambda = 1$</td>
</tr>
<tr>
<td>$\sigma = 0.33$</td>
<td>1.03</td>
<td>1.30</td>
<td>1.02</td>
<td>1.35</td>
<td>0.88</td>
<td>0.11</td>
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<tr>
<td>$\sigma = 0.8$</td>
<td>1.04</td>
<td>1.14</td>
<td>1.04</td>
<td>1.20</td>
<td>1.04</td>
<td>1.14</td>
</tr>
<tr>
<td>$\sigma = 1$</td>
<td><strong>1.04</strong></td>
<td><strong>1.10</strong></td>
<td><strong>1.04</strong></td>
<td><strong>1.15</strong></td>
<td><strong>1.04</strong></td>
<td><strong>1.10</strong></td>
</tr>
<tr>
<td>$\sigma = 1.2$</td>
<td>1.04</td>
<td>1.08</td>
<td>1.04</td>
<td>1.12</td>
<td>1.04</td>
<td>1.08</td>
</tr>
<tr>
<td>$\sigma = 4$</td>
<td>1.05</td>
<td>1.02</td>
<td>1.05</td>
<td>1.01</td>
<td>1.05</td>
<td>1.02</td>
</tr>
<tr>
<td>$\sigma \to \infty$</td>
<td>1.05</td>
<td>0.99</td>
<td>1.05</td>
<td>0.98</td>
<td>1.05</td>
<td>0.99</td>
</tr>
</tbody>
</table>
Figure 1: Dynamic Response to a Permanent Tied Aid Shock
Unitary Elasticity of Substitution ($\sigma = 1$)

1.1. Transitional Adjustment Locus.

1.2. Market Price of Private Capital ($K$).

1.3. Consumption-Private Capital Ratio ($C/K$).

1.4. Consumption-Output Ratio ($C/Y$).

1.5. Debt-Output Ratio ($N/Y$).

1.6. Growth Rates.
Figure 2: Dynamic Response to a Temporary Untied Aid Shock
Unitary Elasticity of Substitution ($\sigma = 1$)

2.1. Transitional Adjustment Locus.

2.2. Market Price of Private Capital (K).

2.3. Consumption-Private Capital Ratio (C/K).

2.4. Consumption-Output Ratio (C/Y).

2.5. Debt-Output Ratio (N/Y).

2.6. Growth Rates
3.1. Transitional Adjustment Locus.

3.2. Market Price of Private Capital (K).

3.3. Consumption-Private Capital Ratio (C/K).

3.4. Consumption-Output Ratio (C/Y).

3.5. Debt-Output Ratio (N/Y).

3.6. Growth Rates.
References

Arrow, K.J. and M. Kurz (1970), *Public Investment, the Rate of Return, and Optimal Fiscal Policy*, Baltimore: Johns Hopkins University Press.


