Econ 8050  
Macroeconomic Theory II  
Assignment 1/Solutions  

1. Suppose that for the last five years, the Federal Reserve Bank has maintained a policy of targeting the unemployment rate. This year, unexpectedly, the Federal Reserve announced a sudden policy shift, from a target rate of unemployment to a target rate of inflation. Explain how the economy would react to this policy shift and discuss the role played by the underlying expectations formation mechanism. [5]  

Answer: If expectations are formed adaptively in this economy, then the Federal Reserve’s sudden policy shift will not be internalized by economic agents, whose forecasts of the future are conditioned by past information about the Fed’s policies (stabilizing unemployment). Therefore, the expected and actual rate of inflation remain unchanged at the time of the policy shift. By contrast, under rational expectations, the behavior of economic agents is forward-looking and therefore the Fed’s policy change represents new information, or “news.” Consequently, expectations of future inflation are instantaneously revised by agents which, due to perfect foresight, leads to an immediate adjustment of the actual rate of inflation.  

2. Refer to the Cagan Monetary Model for this question.  
   (i) Explain the stability condition in this model when expectaions are formed adaptively. Why is such a condition necessary?  
   (ii) When adaptive expectations are stable in this model, prices adjust sluggishly to a change in monetary policy. Explain the intution behind the sluggish adjustment of the price level.  
   (iii) How and why is the adjustment of the price level different when this model is solved using rational expectaions? [2 x 3 = 6]  

Answer: (i) The stability condition in the Cagan model with adaptive expectations depends on the magnitude of two structural parameters: $\alpha$, the semi-elasticity of money demand with respect to inflationary expectations, and $\gamma$, the speed with which the forecast of future expected inflation is changed in response to a current forecast error. The stability condition imposes a restriction on the magnitude of these parameters, i.e., $\alpha \gamma < 1$. This implies a trade-off between $\alpha$ and $\gamma$ in order for expectations to be stabilizing. The intuition can be explained as follows: for any given $\gamma$, a large value of $\alpha$ implies that money demand is very sensitive to small changes in the expected inflation rate. This means that a small change in money supply can generate large changes in money demand which, in turn, can be destabilizing for the money market. A analogous argument applies to $\gamma$ as well.  

(ii) Under adaptive expectations, an unanticipated change in monetary policy has no effect on current expected inflation, since the forecasts of future inflation is backward-looking. Suppose that money supply is reduced by the Central Bank. On impact of the shock, the real supply of money falls, since the price-level is pre-determined and therefore does not respond. In the next period, agents learn about the lower money supply and realize that they had overestimated future inflation. Consequently, the forecast of future inflation is
revised downwards. This leads to a gradual decline in the price level until it reaches the new lower level of money supply.

(iii) When the above model is solved using rational expectations, the reduction in money supply represents new information for economic agents. Since the current price level reflects all available information about current and future money supply, agents instantaneously expect future inflation to fall. Under perfect foresight, this leads to an immediate and full adjustment of the price level.

3. Consider a monetary policy change in an economy characterized by rational expectations. The Central Bank, in order to stabilize the price level, initiates an unanticipated reduction in the quantity of money, from its initial level of \( m \) to \( m/2 \). However, the Central Bank is considering three options with regard to the timing and duration of this change in policy:

(i) The policy change is announced today \((t = 0)\), but will take place only at a future (known) date, \( T > 0 \).

(ii) The policy change is implemented today \((t = 0)\), but is designed to be only temporary and will be reversed at a future date, \( T > 0 \).

(iii) The policy change is implemented immediately and represents a permanent reduction in money supply.

Assuming that the Central Bank’s announcements are credible and agents in the economy have perfect foresight and full information, characterize the dynamic response of the economy under each of the above three policy options. Compare and discuss these responses, explaining the underlying intuition in each case.

\[ [3 \times 3 = 9] \]

**Answer:** The forward-looking solution for the time path of the price level (Sargent and Wallace, 1973) is:

\[
p(t) = \frac{e^{t/\alpha}}{\alpha} \int_{t}^{\infty} m(s)e^{-s/\alpha}ds
\]

(i) The policy change is announced today \((t = 0)\), but will take place only at a future (known) date, \( T > 0 \).

In this example,

\[
m(t) = m, \quad t < T
\]

\[
m(t) = \frac{m}{2}, \quad t \geq T
\]

Using this information in (1), we have

\[
p(t) = \frac{e^{t/\alpha}}{\alpha} \left[ \int_{t}^{T} m e^{-s/\alpha} ds + \int_{T}^{\infty} \frac{m}{2} e^{-s/\alpha} ds \right]
\]
\[ p(t) = \frac{e^{t/\alpha}}{\alpha} \left[ m \int_{t}^{T} e^{-s/\alpha} ds + \frac{m}{2} \int_{T}^{\infty} e^{-s/\alpha} ds \right] \]

\[ \Rightarrow p(t) = m \left[ 1 - \frac{e^{(t-T)/\alpha}}{2} \right] \quad (2) \]

Therefore, on impact of the shock,

\[ p(0) = m \left[ 1 - \frac{e^{-T/\alpha}}{2} \right] \]

The instantaneous jump in the price level is

\[ dp(0) = m \left[ 1 - \frac{e^{-T/\alpha}}{2} \right] - m = -m \frac{e^{-T/\alpha}}{2} < 0 \]

Therefore, on impact, the price level jumps down, but by less than \( m/2 \). In transition, the movement of the price level is given by (2). It can easily be verified that the price level will be falling during transition. finally, when the policy is implemented at \( t = T \),

\[ p(T) = \frac{m}{2} \]

i.e., the adjustment of the price level is complete.

(ii) The policy change is implemented today \( (t = 0) \), but is designed to be only temporary and will be reversed at a future date, \( T > 0 \).

The solution proceeds in a manner analogous to (i) and therefore we can be brief here.

\[ m(t) = \begin{cases} \frac{m}{2} & \text{if } t < T \\ m & \text{if } t \geq T \end{cases} \]

\[ p(t) = \frac{e^{t/\alpha}}{\alpha} \left[ \int_{t}^{T} \frac{m}{2} e^{-s/\alpha} ds + \int_{T}^{\infty} me^{-s/\alpha} ds \right] \]

You should be able to show that following the reduction in money supply, the price level jumps down instantaneously, but by less than \( m/2 \). Following the initial jump, the price level rises over time to reach \( m \) when the initial level of money supply is restored at time \( T \).

(iii) The policy change is implemented immediately and represents a permanent reduction in money supply.

In this case, the price level immediately adjusts to the new, lower level of money supply, i.e., \( p(t) = m/2 \).